

**NONLINEAR STABILITY ANALYSIS OF ELECTRIFIED
LIQUID JET UNDER THE INFLUENCE OF A UNIFORM
AXIAL PERIODIC ELECTRIC FIELD**

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Abstract :

The parametric excitation of surface waves on a liquid jet in the presence of an axial periodic electric field is investigated . The method of multiple scales is used to derive and analyze the necessary and sufficient conditions for stability . Owing to the periodicity , resonant cases appear . Two parametrically nonlinear Schrödinger equations are obtained for the resonance cases . The formula for the surface elevation is derived in each case . A classical nonlinear Schrödinger equation is deduced for the non - resonance case . Investigation of the stability criterion by nonlinear perturbation shows that the periodic electric field has a stabilizing effect .

1. Introduction :

The capillary instability of a circular liquid jet was performed by Rayleigh [1]. He deduced that the maximum growth rate of instability occurs when the wave number $k = 0.697$. A great amount of work has been investigated on jet instability because of its industrial applications. Yuen [2] examined the nonlinear capillary instability of a liquid jet. His analysis revealed that the cutoff wave number which separates stable from unstable disturbances is amplitude dependent. Nayfeh [3] used the method of multiple scales to study the nonlinear capillary stability of a cylindrical column of liquid. Rutland and Jameson [4] studied experimentally the distribution of drop sizes produced by the break up of capillary liquid jets. Melcher [5] suggested the surface waves of the capillary jet under the influence of electric field.

The object of the present paper is to discuss the nonlinear stability of electrified liquid jet under the influence of the influence of an axial periodic electric field.

In the plane geometry, El Shehawey and Abd El - Gawaad [6-8] discussed the instability of an interface between two fluids under the effect of a periodic electric field. El - Dib [9] investigated resonance's of interfacial waves in a nonlinear interfacial instability of two superposed electrified fluids stressed by a time - dependent electric field. The necessary and sufficient conditions for stability are deduced. Mahmoud [10] studied the parametric third - subharmonic resonance instability in nonlinear electrohydrodynamic Rayleigh - Taylor with mass and heat transfer.

In this presentation, we used the method of multiple scales to obtain two parametric nonlinear Schrödinger equations in the resonance cases.

2. Mathematical formulation :

We shall consider liquid jet with radius R . The inner and outer fluids have densities and dielectric constants $\rho^{(1)}$, $\epsilon^{(1)}$ and $\rho^{(2)}$, $\epsilon^{(2)}$ respectively. The fluids are assumed to be inviscid and incompressible. We shall use a cylindrical system of coordinates (r, θ, z) . The gravitational force is neglected.

The system is imbedded in an axial periodic electric field in the direction of z - axis:

$$\underline{E} = (E_0 + \epsilon \hat{E} \cos \omega t) \underline{e}_z, \quad (1)$$

in which E_0 is the electric field intensity, \hat{E} is the amplitude of the periodic term, ϵ is a small dimensionless parameter and \underline{e}_z is the unit vector in the direction of z - axis. In the equilibrium state, we shall assume that there are no surface charges at the interface and therefore the surface charge density will be vanished during the perturbation.

All physical quantities are normalized by using the characteristic length R and the characteristic time $(\rho^{(1)} R^3 / T)^{\frac{1}{2}}$, where T is the surface tension and $\rho^{(1)}$ is the density of the inner fluid.

The basic equations governing the motion are

$$\nabla^2 \phi^{(1)} = \nabla^2 \psi^{(1)} = 0, \quad \text{at } r < 1 + \eta(z, t), \quad (2)$$

$$\nabla^2 \phi^{(2)} = \nabla^2 \psi^{(2)} = 0, \quad \text{at } r = 1 + \eta(z, t), \quad (3)$$

where η denotes the free surface displacement above the equilibrium level, ϕ and ψ are the velocity and electrostatic potentials, respectively.

The boundary conditions at the free surface $r = 1 + \eta(z, t)$ are :

(1) The kinematics boundary condition is :

$$-\frac{\partial \eta}{\partial t} + \frac{\partial \phi^{(1),(2)}}{\partial r} - \frac{\partial \eta}{\partial z} \frac{\partial \phi^{(1),(2)}}{\partial z} = 0, \quad (4)$$

(2) The tangential component of the electric field should be continuous at the interface :

$$[[\tilde{\varepsilon} \frac{\partial \psi}{\partial z}]] + [[\frac{\partial \psi}{\partial r}]] \frac{\partial \eta}{\partial z} = 0, \quad (5)$$

where $[[\]]$ represents the jump across the interface.

(3) The normal electric displacement is continuous at the interface :

$$[[\tilde{\varepsilon} \frac{\partial \psi}{\partial r}]] + E_0 [[\tilde{\varepsilon}]] \frac{\partial \eta}{\partial z} + \varepsilon \hat{E} [[\tilde{\varepsilon}]] \frac{\partial \eta}{\partial z} - \frac{\partial \eta}{\partial z} [[\tilde{\varepsilon} \frac{\partial \psi}{\partial z}]] = 0, \quad (6)$$

(4) The normal hydrodynamic stress is balanced by the normal electric stress. The balance condition is then :

$$\begin{aligned} & \frac{\partial \phi^{(1)}}{\partial t} - \rho \frac{\partial \phi^{(2)}}{\partial t} + \frac{1}{2} \left[\left(\frac{\partial \phi^{(1)}}{\partial r} \right)^2 - \rho \left(\frac{\partial \phi^{(2)}}{\partial r} \right)^2 \right] + \frac{1}{2} \left[\left(\frac{\partial \phi^{(1)}}{\partial z} \right)^2 - \rho \left(\frac{\partial \phi^{(2)}}{\partial z} \right)^2 \right] = \frac{\partial^2 \eta}{\partial z^2} + \eta \\ & - \eta^2 + \frac{1}{2} \left(\frac{\partial \eta}{\partial z} \right)^2 - \frac{1}{2} \eta \left(\frac{\partial \eta}{\partial z} \right)^2 - \frac{3}{2} \frac{\partial^2 \eta}{\partial z^2} \left(\frac{\partial \eta}{\partial z} \right)^2 - \frac{1}{2} [[\left(\frac{\partial \psi}{\partial r} \right)^2]] - (E_0 + \varepsilon \hat{E} \cos \omega t) [[\tilde{\varepsilon} \frac{\partial \psi}{\partial z}]] \\ & + \frac{1}{2} [[\tilde{\varepsilon} \left(\frac{\partial \psi}{\partial z} \right)^2]] + (E_0 + \varepsilon \hat{E} \cos \omega t) \{ -[[\tilde{\varepsilon}]] (E_0 + \varepsilon \hat{E} \cos \omega t) \left(\frac{\partial \eta}{\partial z} \right)^2 + 2 [[\tilde{\varepsilon} \left(\frac{\partial \psi}{\partial z} \right)]] \left(\frac{\partial \eta}{\partial z} \right)^2 \\ & - 2 \frac{\partial \eta}{\partial z} [[\tilde{\varepsilon} \frac{\partial \psi}{\partial r}]] \} + 2 \frac{\partial \eta}{\partial z} [[\tilde{\varepsilon} \frac{\partial \psi}{\partial r} \frac{\partial \psi}{\partial z}]], \end{aligned} \quad (7)$$

where $\rho = \rho^{(2)} / \rho^{(1)}$.

We employ the method of multiple scales [11] to discuss the nonlinear stability problem posed by the equations (1)-(7). We expand the various variables in ascending powers in terms of a small dimensionless parameter ε characterizing the steepness ratio of the wave. The independent variables z, t are scaled in a like manner,

$$Z_n = \varepsilon^n z, \quad T_n = \varepsilon^n t, \quad n=0,1,2,3, \quad (8)$$

and the variables may be expanded as

$$\eta(z, t) = \sum_{n=1}^3 \varepsilon^n \eta_n(Z_0, Z_1, Z_2, T_0, T_1, T_2) + o(\varepsilon^4), \quad (9)$$

$$\phi^{(1),(2)}(r, z, t) = \sum_{n=1}^3 \varepsilon^n \phi_n^{(1),(2)}(r, Z_0, Z_1, Z_2, T_0, T_1, T_2) + o(\varepsilon^4), \quad (10)$$

$$\psi^{(1),(2)}(r, z, t) = \sum_{n=1}^3 \varepsilon^n \psi_n^{(1),(2)}(r, Z_0, Z_1, Z_2, T_0, T_1, T_2) + o(\varepsilon^4). \quad (11)$$

Nayfeh [12] perturbed the surface deflection $\eta(z, t)$ in the form

$$\eta = \varepsilon \eta_1 + \varepsilon^2 \eta_2 + \varepsilon^3 \eta_3 + \dots \quad (12)$$

Because of the complexity of problems involving periodic terms, El-Dib [9] expanded every order of the above surface deflection in the form

$$\eta_K = \eta_{K0} + \varepsilon \eta_{K1} + \varepsilon^2 \eta_{K2} + \dots (K=1,2,\dots), \quad (13)$$

where

$$\eta_1 = \eta_{10}, \quad \eta_2 = \eta_{20} + \eta_{11}, \quad \eta_3 = \eta_{30} + \eta_{21} + \eta_{12}, \quad (14)$$

where $\varepsilon \eta_{10} + \varepsilon^2 \eta_{20} + \varepsilon^3 \eta_{30} + \dots$ denotes the surface deflection in the absence of the periodic electric field. We expand the quantities involved the boundary conditions in Taylor's series about $r = 1$. Substituting from (8)–(11) into (2)–(7), and equating the coefficient of like power of ε , we get the linear as well as successive higher-order equations. The hierarchy of the equations for each order can be deduced with the knowledge of solutions of the previous orders.

3. Linear theory:

The surface deflection in the first order problem η_1 is given by

$$\eta_1 = \mu(Z_1, Z_2, T_0, T_1, T_2) e^{ikz_0} + \text{C.C.}, \quad (15)$$

in which k is the wave number and C.C. is the complex conjugate. The solution of the first order problem is due to the linear differential equation which is satisfied the unknown function μ . This equation is given by

$$\frac{\partial^2 \mu}{\partial T_0^2} + \frac{k I_1(k) K_1(k)}{\rho^*} \left[k^2 - 1 + \frac{k I_0(k) K_0(k)}{\varepsilon^*(k)} E_0^2 (\tilde{\varepsilon}^{(1)} - \tilde{\varepsilon}^{(2)})^2 \right] \mu = 0, \quad (16)$$

where

$$\rho^* = I_0(k) K_1(k) + \rho K_0(k) I_1(k),$$

$$\varepsilon^*(k) = \tilde{\varepsilon}^{(1)} I_1(k) K_0(k) + \tilde{\varepsilon}^{(2)} K_1(k) I_0(k).$$

Equation (16) admits the following solution:

$$\mu(Z_1, Z_2, T_0, T_1, T_2) = A(Z_1, Z_2, T_1, T_2) e^{-i\omega_0 T_0} + \text{C.C.}, \quad (17)$$

in which ω_0 is the frequency of the disturbance and A is the amplitude of the propagation wave and will be obtained later by the solvability conditions. The dispersion relation for the linearized problem is given by:

$$\omega_0^2 = \frac{k I_1(k) K_1(k)}{\rho^*} \left[k^2 - 1 + \frac{k I_0(k) K_0(k)}{\varepsilon^*(k)} E_0^2 (\tilde{\varepsilon}^{(1)} - \tilde{\varepsilon}^{(2)})^2 \right], \quad (18)$$

where I_0 , I_1 , K_0 and K_1 are the modified Bessel functions. This dispersion relation is obtained by Nayyar and Murty [13]. It is shown from the above equation that the constant electric field has a stabilizing effect on the wave motion. We can deduce that the periodic electric field has no effect in the first order problem. In order to study the amplitude modulation, we consider $\omega^2 > 0$ and proceed to the second order.

4. Second order problem:

By substituting the solution of the first order problem into the second order, we get two cases. The first case is the non-resonance case, when the disturbance

frequency ω_0 is away from the field frequency ω . The second one is the resonance case, which produces when ω is approaching $2\omega_0$.

The solvability condition in the non- - resonance case is given by:

$$\frac{\partial A}{\partial T_1} + \frac{d\omega_0}{dk} \frac{\partial A}{\partial Z_1} = 0, \quad (19)$$

In the resonant case, we introduce the detuning parameter σ to describe the nearness of ω to $2\omega_0$. Thus, we can write

$$\omega = 2\omega_0 + 2\varepsilon\sigma. \quad (20)$$

The solvability condition in the resonant case is given by:

$$\frac{\partial A}{\partial T_1} + \frac{d\omega_0}{dk} \frac{\partial A}{\partial Z_1} = i\alpha_0 \bar{A} e^{-2i\sigma T_0}, \quad (21)$$

where

$$\alpha_0 = -\frac{k^2(\tilde{\varepsilon}^{(1)} - \tilde{\varepsilon}^{(2)})^2}{2\omega_0 \rho^* \varepsilon^*(k)} E_0 \hat{E} I_1(k) K_1(k) I_0(k) K_0(k). \quad (22)$$

The uniformly valid second - order elevation can be written as:

$$\eta_2 = 2S_1 A \bar{A} + \gamma_1^* e^{ikZ_0} + \Omega_2 A^2 e^{2i(kZ_0 - \omega_0 T_0)} + C.C. \quad (23)$$

where

$$\gamma_1^* = \frac{k^2(\tilde{\varepsilon}^{(1)} - \tilde{\varepsilon}^{(2)})^2 E_0 \hat{E}}{\omega(\omega + 2\omega_0) \rho^* \varepsilon^*(k)} A I_1(k) K_1(k) I_0(k) K_0(k) e^{-i(\omega + \omega_0) T_0} + C.C., \quad (24)$$

S_1 and Ω_2 are given in the appendix. We proceed to the third order problem to obtain the equation for the evolution of travelling waves.

5. Third order problem:

We substitute the first and second order solutions into the third order equations. In order to study this order, we must distinguish between two cases: the first case is the non- - resonance case, when ω is not near ω_0 and $2\omega_0$. The second one is the resonance case which arises when ω is approaching $2\omega_0$ or ω_0 .

(5.1) The non - resonance case:

By using (19), the solvability condition in the non- - resonance case is due to a nonlinear Schrödinger equation that can be written as:

$$iR_3 \frac{\partial A}{\partial T_2} + iR_2 \frac{\partial A}{\partial Z_2} + \frac{1}{2} \frac{d^2 \omega_0}{dk^2} \frac{\partial^2 A}{\partial Z_1^2} + R_4 A^2 \bar{A} + R_1 A = 0, \quad (25)$$

where R_1 , R_2 , R_3 and R_4 are lengthy and not included here. They are available from the author.

We introduce the transformation:

$$\xi = \varepsilon(Z - \frac{R_2}{R_1}t) \quad \text{and} \quad \tau = \varepsilon^2 t, \quad (26)$$

Substituting from (8) and (26) into (25), we have

$$iR_3 \frac{\partial A}{\partial \tau} + \frac{1}{2} \frac{d^2 \omega_0}{dk^2} \frac{\partial^2 A}{\partial \xi^2} + R_4 A^2 \bar{A} + R_1 A = 0. \quad (27)$$

The solvability condition of the nonlinear Schrödinger (27), is given by

$$R_4 \frac{d^2 \omega_0}{dk^2} < 0. \quad (28)$$

This condition has been obtained by Hasimoto & Ono [14], Nayfeh [12] and Mohamed & ElShehawey [15-18]. Mohamed and ElShehawey [18] studied the electrohydrodynamic stability of a horizontal interface separating two dielectric streaming fluids subjected to a normal electric field. Based on the method of multiple scales, two nonlinear Schrödinger equations describing the perturbed system are derived. One of them is used to get the electrohydrodynamic nonlinear cut-off wavenumber separating stable and unstable disturbances while the other equation is used to obtain the necessary condition for stability and instability for the system. It is noted that both electric field and streaming play a dual role on the stability analysis.

5.2 The resonance cases:

5.2.1 The case of ω near ω_0 :

In the resonant case when $\omega \approx \omega_0$, we introduce a detuning parameter

σ_2 defined by

$$\omega = \omega_0 + \varepsilon^2 \sigma_2. \quad (29)$$

In this case, the solvability condition is due to the following a parametric nonlinear Schrödinger equation

$$iR_3 \frac{\partial A}{\partial \tau} + \frac{1}{2} \frac{d^2 \omega_0}{dk^2} \frac{\partial^2 A}{\partial \xi^2} + R_4 A^2 \bar{A} + R_1 A + R_8 A e^{-2i\sigma_2 \tau} = 0, \quad (30)$$

where R_8 is lengthy and not included here. It is available from the author. To discuss the stability conditions, we follow El - Dib's [19] analysis. We assume that (30) admits the time dependent solution:

$$A = m_1 e^{-i\sigma_2 \tau}, \quad (31)$$

where m_1 is a real amplitude is given by

$$m_1^2 = -\frac{1}{R_4} (\sigma_2 R_3 + R_1 + R_8). \quad (32)$$

The right hand side of (32) must be positive, this is requires that

$$R_4 (\sigma_2 R_3 + R_1 + R_8) < 0. \quad (33)$$

To obtain the stability criterion, we perturb the solution (31) according to

$$A = [m_1 + \alpha_1(\xi, \tau) + i\beta_1(\xi, \tau)] e^{-i\sigma_2 \tau}, \quad (34)$$

where α_1 and β_1 are real functions. Substituting from (34) into (30), neglecting nonlinear terms in α_1 and β_1 , we obtain

$$R_3 \frac{\partial \alpha_1}{\partial \tau} + \frac{1}{2} \frac{d^2 \omega_0}{dk^2} \frac{\partial^2 \beta_1}{\partial \xi^2} - 2R_8 \beta_1 = 0, \quad (35)$$

$$-R_3 \frac{\partial \beta_1}{\partial \tau} + \frac{1}{2} \frac{d^2 \omega_0}{dk^2} \frac{\partial^2 \alpha_1}{\partial \xi^2} + 2R_4 m^2 \alpha_1 = 0. \quad (36)$$

Since (35) and (36) are linear equations, then their solution can be written in the form

$$\alpha_1(\xi, \tau) = \alpha_1^* e^{i(q\xi - \Omega\tau)}, \quad (37)$$

$$\beta_1(\xi, \tau) = \beta_1^* e^{i(q\xi - \Omega\tau)}, \quad (38)$$

where q and Ω are the wavenumber and frequency satisfy the dispersion relation

$$\Omega^2 = \frac{1}{R_3} \left[\frac{1}{4} q^4 \left(\frac{d^2 \omega_0}{dk^2} \right)^2 + q^2 \left(\frac{d^2 \omega_0}{dk^2} \right) (R_8 - m^2 R_4) - 4R_8 R_4 m^2 \right], \quad (39)$$

It is clear from equation (39) that the necessary and sufficient condition for stability requires that

$$\left[q^4 \left(\frac{d^2 \omega_0}{dk^2} \right)^2 + 4q^2 \left(\frac{d^2 \omega_0}{dk^2} \right) (R_8 - m^2 R_4) - 16R_8 R_4 m^2 \right] > 0. \quad (40)$$

The relation (40) can be written in the form

$$(q^2 - q_1)(q^2 - q_2) > 0, \quad (41)$$

The transition curves separating stable region from unstable region corresponding to

$$q_1 = [1 / \left(\frac{d^2 \omega_0}{dk^2} \right)] (\sigma_2 R_3 + R_1 + R_8), \quad (42)$$

$$q_2 = -[4R_8 / \left(\frac{d^2 \omega_0}{dk^2} \right)]. \quad (43)$$

In this case, the surface deflection is obtained as:

$$\eta = m_1 \varepsilon e^{i[kz - (\omega_0 + \sigma_2 \varepsilon^2)t]} \left[1 + \frac{2\varepsilon k^2 E_0 \hat{E} (\tilde{\varepsilon}^{(1)} - \tilde{\varepsilon}^{(2)})}{\rho^* \varepsilon^* (k) \omega (\omega^2 - 4\omega_0^2)} I_1(k) K_1(k) I_0(k) K_0(k) x \right]$$

$$x(\omega \cos \omega t + 2i\omega_0 \sin \omega t) + \varepsilon^2 \Omega_2 m_1^2 e^{2i[kz - (\omega_0 + \sigma_2 \varepsilon^2)t]} + \dots \quad (44)$$

The transition curves (42) and (43) are plotted in the $(q^2 - k)$ plane for a sample case. Figure (1) is for a system with $\rho^{(1)} = 0.99823 \text{ gm/cm}^3$, $\rho^{(2)} = 0.0013 \text{ gm/cm}^3$, $\tilde{\varepsilon}^{(1)} = 80.08$, $\tilde{\varepsilon}^{(2)} = 1.00059$, $E_0 = 1 \text{ dynes / csu}$, $\hat{E} = 0.2 \text{ dynes / csu}$ and $\varepsilon = 0.1$. The region in the $(q^2 - k)$ plane indicated by S represent situation where every point in the region satisfies the inequalities (33) and (41). The unstable region are labeled by the symbols u_1 and u_2 . In figure (1), the curves which are labeled by x , o and Δ represent the external frequency $\omega = 0.9749$, 1.2 and 1.6 Hz respectively. It is shown that the stable region is always enlarged as the external frequency is decreased. Thus, the external frequency has a destabilizing effect. El - Dib [9] obtained the same result in the plane geometry. El Dabe et al [20] studied the linear electrohydrodynamic stability of two cylindrical interface under the influence of a tangential periodic electric field. They

obtained two simultaneous ordinary differential equations of the Mathieu type. It is shown that the constant tangential field has a stabilizing effect while the tangential periodic field has a stabilizing influence except at resonance points. The thickness of the jet plays a role in the stability criterion and the external frequency is used to control the position of the resonance regions. The special cases of large modulation and small modulation are also studied. It is shown that for large modulation the electric field has destabilizing influence.

Figure (2) represents the same system considered in figure (1), but $\omega = 0.9749$ Hz, the curves which are marked by the symbols \times , \circ and Δ represent $E_0 = 1, 1.2$ and 1.4 dynes / esu. respectively. It is found that the increase of the constant electric field increases the unstable region u_2 , while the unstable region u_1 . Therefore, the constant electric field plays a dual role in the stability criteria. In the plane geometry, El - Dib [9] found that the constant electric field plays a dual role in the stability analysis. Mohamed et al [17] studied the nonlinear electrohydrodynamic stability of a horizontal interface separating two dielectric fluids are stressed by a tangential electric field. Based on the method of multiple scales, two nonlinear Schrödinger equations are obtained.

The electrohydrodynamic cutoff wavenumber separating stable and unstable disturbances is calculated. It can be seen that if a finite amplitude disturbance is stable, then a small modulation to the wave is also stable. It is concluded that the tangential field plays a dual role in the stability criterion. Also, it is noted that the field is stabilizing or destabilizing according to whether the lower fluid has a larger or smaller dielectric constant than the upper one.

Figure (3) represents the same system considered in figure (1), but $\omega = 0.9749$ Hz. and $E_0 = 1$ dynes / esu, the curves which are indicated by the symbols \times , \circ and Δ represent $\hat{E} = 0.2, 0.22$ and 0.25 dynes/esu. respectively. It is concluded that the increase in the amplitude of the electric field increases the size of the stable region. Therefore, the amplitude of the periodic electric field has stabilizing effect. Malik and Singh [21] studied the nonlinear waves on the surface of a magnetic fluid jet. It is observed that the wavetrain of constant amplitude is unstable against modulations. As the magnetic field is increased, the wavenumber at which the modulation instability sets in shifts into the region of long wave length.

5.2.2 The case of ω near $2\omega_0$:

In the resonant case when $\omega \approx 2\omega_0 + 2\varepsilon\sigma$, the solvability condition is due to a parametric nonlinear Schrödinger equation:

$$iR_3 \frac{\partial A}{\partial \tau} + \frac{1}{2} \frac{d^2 \omega_0}{dk^2} \frac{\partial^2 A}{\partial \xi^2} + R_4 A^2 \bar{A} + R_6 A + (iR_5 \frac{\partial \bar{A}}{\partial \xi} + R_7 \bar{A}) e^{-2i\sigma \varepsilon^{-1} \tau} = 0, \quad (45)$$

where R_5 , R_6 and R_7 are lengthy and not included here. They are available from the author. The conditions of the stability of equation (45) are given by [19]:

$$R_4 \left(\frac{R_3 \sigma}{\varepsilon} + R_6 + R_7 \right) < 0, \quad (46)$$

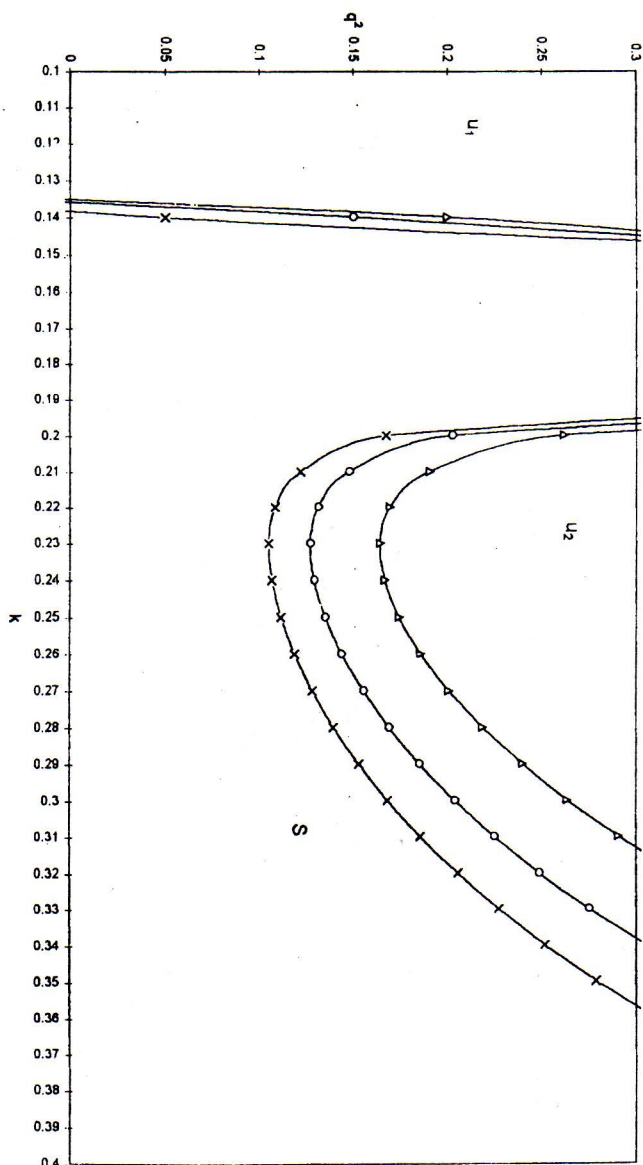


Figure (1) : Represents the plane $(q^2 - k)$ for a system $\rho^{(1)} = 0.99823 \text{ gm/cm}^3$, $\rho^{(2)} = 0.0013 \text{ gm/cm}^3$, $\tilde{\epsilon}^{(1)} = 80.08$, $\tilde{\epsilon}^{(2)} = 1.00059$, $E_0 = 1 \text{ dynes/esu}$, $\hat{E} = 0.2 \text{ dynes/esu}$ and $\epsilon = 0.1$. The curves which are labeled by x , o and Δ represent the external frequency $\omega = 0.9749, 1.2$ and 1.6 Hz respectively.

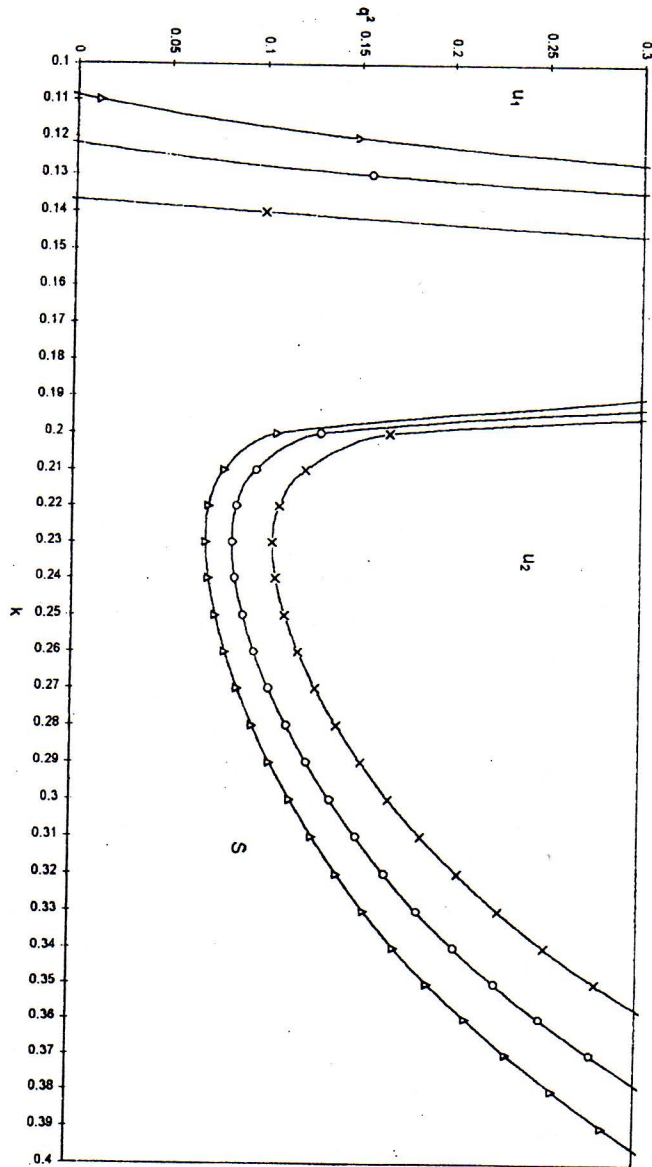


Figure (2): Represents the same system considered in figure (1), but $\omega = 0.9749$ Hz, the curves which are marked by the symbols x , o and Δ represent $E_0 = 1$, 1.2 and 1.4 dynes / esu . respectively .

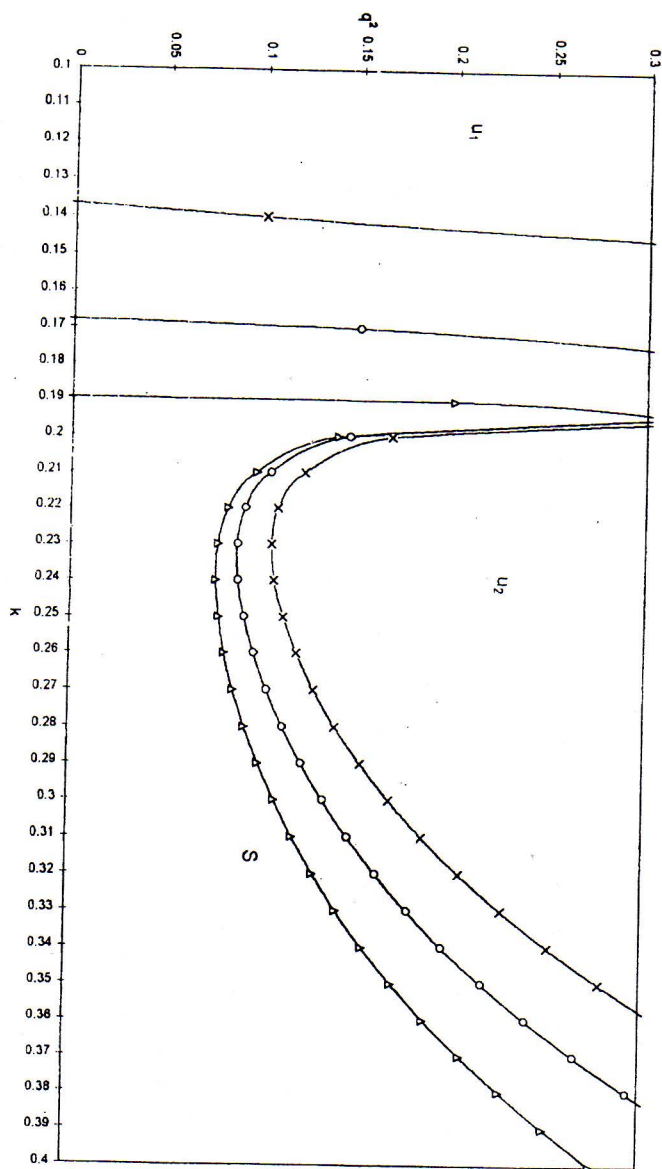


Figure (3) :Represents the same system considered in figure (1) , but $\omega = 0.9749$ Hz. and $E_0 = 1$ dynes /esu , the curves which are indicated by the symbols x , o and Δ represent $\hat{E} = 0.2$, 0.22 and 0.25 dynes/esu . respectively .

$$R_5^2 + \left(\frac{R_3\sigma}{\varepsilon} + R_6 + 2R_7 \right) \frac{d^2\omega_0}{dk^2} > 0, \quad (47)$$

$$R_7 \left(\frac{R_3\sigma}{\varepsilon} + R_6 + 2R_7 \right) > 0. \quad (48)$$

The transition curves separating stable region from unstable region corresponding to

$$\left(\frac{R_3\sigma}{\varepsilon} + R_6 + R_7 \right) = 0, \quad (49)$$

$$R_4 = 0, \quad (50)$$

$$R_5^2 + \left(\frac{R_3\sigma}{\varepsilon} + R_6 + 2R_7 \right) \frac{d^2\omega_0}{dk^2} = 0, \quad (51)$$

$$R_7 = 0. \quad (52)$$

Equation (49) and (51) can be written in the form

$$R_6'E^{*2} + R_7'E^* + \frac{R_3(\omega - 2\omega_0)}{2} = 0 \quad (53)$$

$$(R_5'^2 + R_6' \frac{d^2\omega_0}{dk^2})E^{*2} + 2R_7' \frac{d^2\omega_0}{dk^2} E^* + \frac{R_3(\omega - 2\omega_0)}{2} \frac{d^2\omega_0}{dk^2} = 0, \quad (54)$$

where $E^* = \varepsilon \hat{E}$, $R_5 = R_5' \hat{E}$, $R_6 = R_6' \hat{E}$ and $R_7 = R_7' \hat{E}$.

In this case, the surface deflection is given by

$$\eta = m\varepsilon e^{i[kz - (\omega_0 + \sigma\varepsilon)t]} \left[1 + \frac{\varepsilon k^2 E_0 \hat{E}(\tilde{\varepsilon}^{(1)} - \tilde{\varepsilon}^{(2)})}{\rho^* \varepsilon^*(k) \omega(\omega + 4\omega_0)} I_1(k) K_1(k) I_0(k) K_0(k) \right] + \varepsilon^2 \Omega_2 m^2 e^{2i[kz - (\omega_0 + \sigma\varepsilon)t]} + \dots \quad (55)$$

In the case of $\omega \approx 2\omega_0 + 2\varepsilon\sigma$, the transition curves (53) and (54) are plotted in the $(E^* - k)$ plane for a sample case. The region in the $(E^* - k)$ plane labeled by S represent the situation where every point in the region satisfies the inequalities (46) - (48). Figure (4) represents the same system considered in figure (1), but $E_0 = 1$ dynes/csu, the curves which are indicated by the symbols x, o and Δ represent external frequency $\omega = 0.4$, 0.5 and 0.8 Hz. respectively. The dotted line ($\omega = 2\omega_0$) divided the plane $(E^* - k)$ into two regions. The right region satisfies the case of $\omega < 2\omega_0$. The left region for the values of $\omega > 2\omega_0$. It is noted that the external frequency has a stabilizing effect. Also, it is found that the resonance point $k^*(\omega = 2\omega_0)$ is shifted to the right hand side in the plane $(E^* - k)$. Mahmoud [10] studied the nonlinear electrohydrodynamic Rayleigh - Taylor instability with mass and heat transfer. The fluids are subjected to a periodic acceleration and a normal electric field. The necessary and sufficient conditions for stability are obtained. It is shown that the thickness of the fluid, the normal electric field and the coefficient of mass and heat transfer have destabilizing effect. Also, it can be seen that the external frequency plays a dual role in the stability analysis. Elhefnawy et al [22] investigated weakly nonlinear stability of interfacial waves propagating between two electrified inviscid fluids stressed by a vertical periodic forcing and a constant horizontal electric field in the presence of mass

and heat transfer. It is observed that the external frequency and the electric field play dual role in the stability criterion.

Figure (5) represents the same system considered in figure (4), but $\omega = 0.8$ Hz, the curves which are labeled by the symbols \times , \circ and Δ represent $E_0 = 1, 1.5$ and 2 dynes/esu respectively. It is shown that the constant field has a destabilizing effect. Also, it is noted that the variation of E_0 with fixed ω results in a shift in the resonance point to the left side. In the plane geometry, El - Dib [19] discussed the effect of a periodic acceleration on nonlinear modulation of interfacial gravity - capillary waves between two electrified fluids under the influence of a horizontal electric field. It is reported that the horizontal electric field plays a dual role in the resonance case.

6. Conclusion:

We have investigated the parametric excitation of nonlinear surface waves of electrified liquid jet subjected to a uniform axial periodic electric field. Two parametric nonlinear Schrödinger equations are obtained in the resonance cases and a classical nonlinear Schrödinger equation in the non - resonance case. In the resonance case of the external frequency ω near the disturbance frequency ω_0 , it is found the constant electric field plays a dual role in the stability analysis and the external frequency has destabilizing effect, while the amplitude of the periodic electric field has a stabilizing effect. For $\omega \approx 2\omega_0 + 2\varepsilon\sigma$, it is shown that the external frequency has a stabilizing effect while the constant electric field has a destabilizing effect.

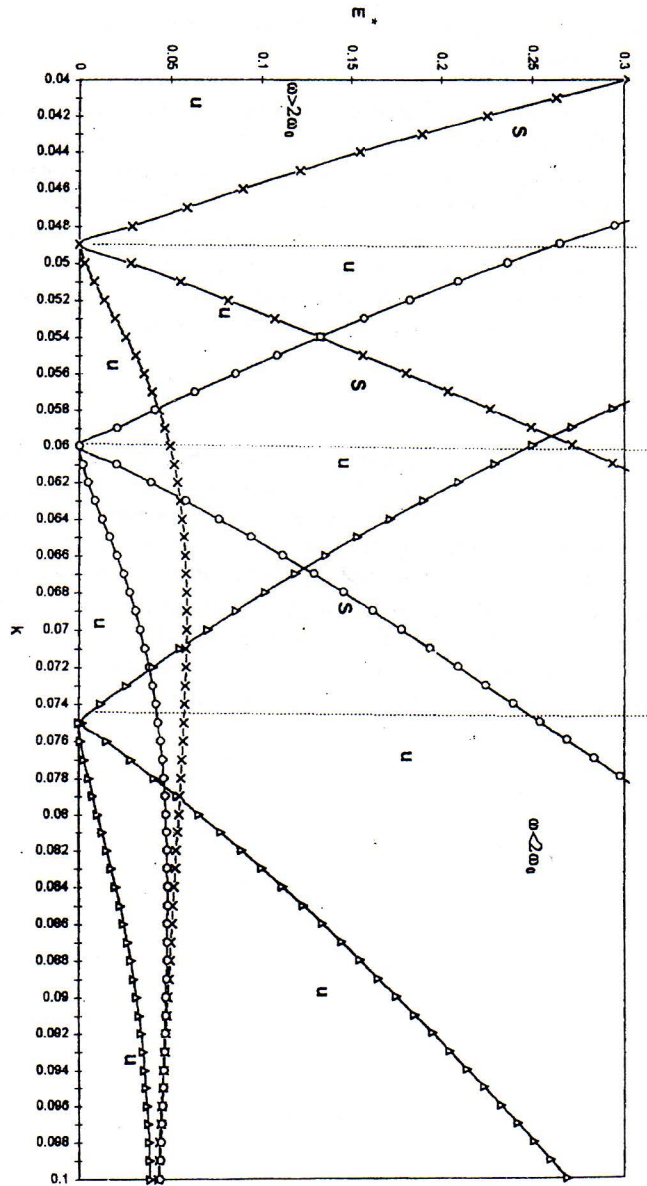


Figure (4): Represents the same system considered in figure (1), but $E_0 = 1$ dynes/esu, the curves which are indicated by the symbols x , o and Δ represent external frequency $\omega = 0.4$, 0.5 and 0.8 Hz. respectively.

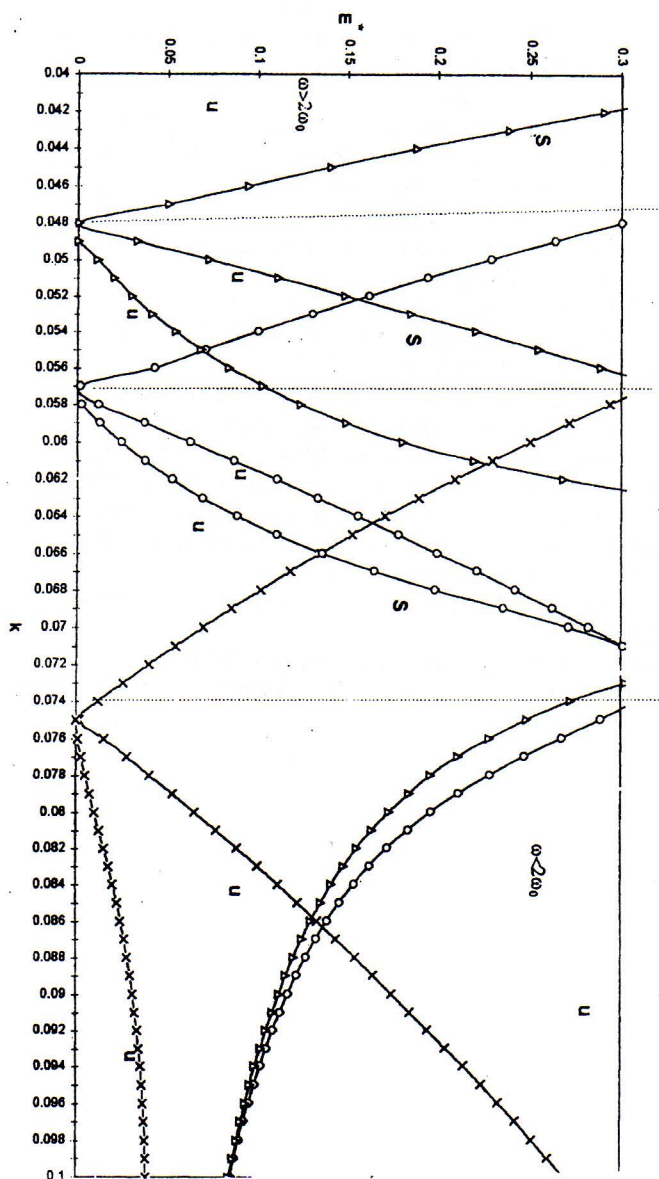


Figure (5): Represents the same system considered in figure (4), but $\omega = 0.8$ Hz, the curves which are labelled by the symbols x, o and Δ represent $E_0 = 1, 1.5$ and 2 dynes/esu .respectively .

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Appendix

$$\begin{aligned}
S_1 = & -\frac{\omega_0^2}{2} \left[1 - \frac{I_0^2(k)}{I_1^2(k)} \right] + \frac{\omega_0^2 \rho}{2} \left[1 - \frac{K_0^2(k)}{K_1^2(k)} \right] - \frac{1}{2} k^2 + 1 + \frac{k^2 E_0^2 (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)})^2}{2 \epsilon^{*2}(k)} x \\
& \times [\tilde{\epsilon}^{(1)} I_1^2(k) K_0^2(k) - \tilde{\epsilon}^{(2)} I_0^2(k) K_1^2(k) - (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)}) I_0^2(k) K_0^2(k)], \\
\Omega_2 = & \left[\frac{3}{2} \omega_0^2 (1 - \rho) + \frac{\omega_0^2}{2 I_1^2(k) K_1^2(k)} \{-I_0^2(k) K_1^2(k) + \rho I_1^2(k) K_0^2(k)\} - 1 - \frac{1}{2} k^2 \right. \\
& + \frac{k E_0^2 (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)})}{\epsilon^*(2k) \epsilon^*(k)} [\epsilon^*(k) \tilde{\epsilon}^{(1)} \tilde{\epsilon}^{(2)} \{I_0(2k) K_1(2k) + I_1(2k) K_0(2k)\} \{I_1(k) K_0(k) + \\
& + I_0(k) K_1(k) + 2 \epsilon^*(k) (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)})^2 I_0(2k) K_0(2k) I_0(k) K_0(k) - \frac{(\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)})}{k} x \\
& \times \epsilon^{*2}(k) I_0(2k) K_0(2k) + \frac{1}{2} \epsilon^*(2k) (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)}) \{\tilde{\epsilon}^{(1)} I_1^2(k) K_0^2(k) - \tilde{\epsilon}^{(2)} I_0^2(k) K_0^2(k)\} \\
& \left. + \frac{1}{2} \epsilon^*(2k) (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)})^2 I_0^2(k) K_0^2(k) - 2 \epsilon^*(2k) \epsilon^{*2}(k) \right] / \left[\frac{-2 \omega_0^2}{k I_1(2k) K_1(2k)} [I_0(2k) x \right. \\
& \left. K_1(2k) - \rho K_0(2k) I_1(2k)] + 4 k^2 - 1 + \frac{2 k E_0^2}{\epsilon^*(2k)} K_0(2k) I_0(2k) (\tilde{\epsilon}^{(1)} - \tilde{\epsilon}^{(2)})^2 \right]
\end{aligned}$$

where

$$\epsilon^*(2k) = \tilde{\epsilon}^{(1)} I_1(2k) K_0(2k) + \tilde{\epsilon}^{(2)} I_0(2k) K_1(2k).$$