

# **NUMERICAL MODELING OF INTERNAL STRESS IN THE SURFACE LAYER OF STEEL SUBJECTED TO SURFACE INDUCTION HARDENING**

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## **1. Introduction**

A result of mating friction elements is a tribological wear, understood as a destruction process consisting in material removal from the surface layer of a solid resulting in a continuous change of shape and size of the friction elements.

As far as terminological wear is concerned, operational durability of both machinery elements and tools can be significantly extended when an appropriately formed surface layer is applied. A function of this layer is to counteract the disadvantageous effects accompanying element's performance. In the case of heat treated surface layers, such as nitrided, carburized or surface hardened layers, it is an internal stress that plays a predominant role in this process. Most of the time it is a compressive stress, since compressive stress contributes to a substantial increase of fatigue strength of machinery elements both in their volume and in the case of contact, impact or heat loads of a cyclic character. Therefore, a significant element of a surface layer characteristics is this layer's thickness, its hardness and a distribution of its internal stress in particular. A collection of these parameters for different surface treatment processes constitutes a specific data base enabling a selection of an optimum (from both viewpoints, technological and economical) formation method of a hardened surface layer, most advantageous for given operational conditions.

A development of computer technology has created a situation when most of the design and construction work of today is based on computer modeling and simulation. Due to this technology optimum design solutions, taking into account real mechanical and heat loads, of many technological problems become possible.

The aim of the present work is to use numerical methods to construct a model of surface layer internal stress. On a basis of numerical analysis this model should enable an estimation of distribution of internal stress in the surface layer of steel resulting from its surface induction hardening. The model will be verified by means of internal stress measurements with the help of the Waisman-Phillips method [1].

## **2. Construction of a numerical model**

A model of internal stress formation as a result of surface hardening of steel was constructed with the help of ANSYS 5.5 software using a finite elements method (FEM) [2]. In order to develop such a FEM model one needs to have material data. As far as the discussed case is concerned, a knowledge of the following material parameters as a function of temperature is necessary: Young modulus  $E$ , linear heat expansion coefficient  $\alpha$ , specific heat  $c$ , thermal conductivity coefficient  $\lambda$ , yield point  $R_e$ .

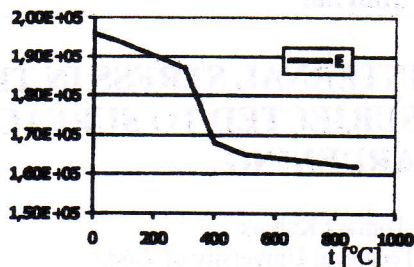
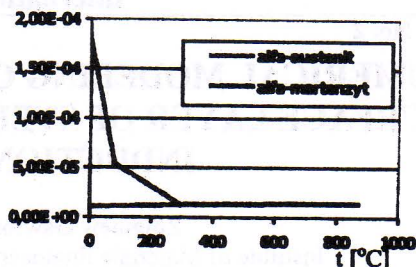
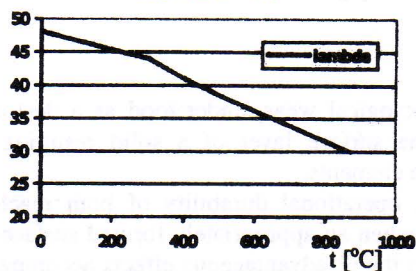
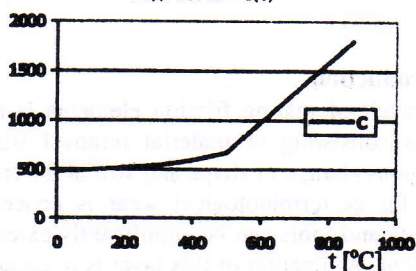
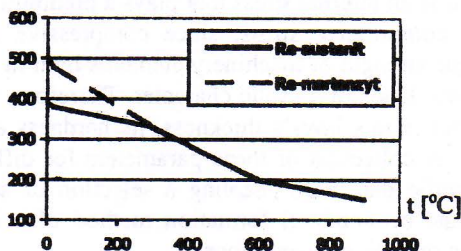
Fig. 1a.  $E = f(t)$ Fig. 1b.  $\alpha = f(t)$ Fig. 1c.  $\lambda = f(t)$ Fig. 1d.  $c = f(t)$ Fig. 1e.  $R_e = f(t)$ 

Figure 1. Temperature dependent parameters assumed in this work [3, 4, 5, 6].

On the basis of literature data [3, 4, 5, 6], the following temperature dependent parameters were assumed for steel of carbon content amounting to 0.5 – 0.6 %:

- Young modulus  $E$  [MPa] – see Figure 1a,
- linear heat expansion coefficient  $\alpha$  [ $K^{-1}$ ] – see Figure 1b,
- thermal conductivity coefficient  $\lambda$  [ $W/m \cdot K$ ] – see Figure 1c,
- specific heat  $c$  [ $J/kg \cdot K$ ] – see Figure 1d,
- yield point  $R_e$  [MPa] – see Figure 1e.

The data found in the literature concern the temperature range of either 0-500°C or 0-600°C. Since knowledge of these parameters for the entire range of 0-900° is necessary, a linear approximation of the data was carried out for these parts of the functions that were unknown.

Remaining material parameters were assumed to be temperature independent: Poisson's number  $\nu = 0.3$  and density  $\rho = 7850 \text{ kg/m}^3$ . The stress – strain characteristics of the material was assumed on the basis of the Prandtl-Reuss plastic flow theory with the kinematic bilinear amplification.



In the numerical model developed with the help of the finite elements method the same elements as those used to solve coupled field problem were applied and "transient" numerical analysis with the integration over time was carried out. For the discrete model, plane tetranodal elements (PLANE 13), with three degrees of freedom (two translational degrees plus temperature) at each node, were used.

The discrete model formulated above was subjected to a load at two stages. At the first stage, of the duration equal  $t_{\text{heating}}$  [s], the model was subjected to a load by applying boundary conditions in the form of node temperature in the nodes corresponding to the sample's surface equal maximum temperature on the surface of the hardened sample. The second stage consists in the cooling of the sample to the room temperature within the (determined empirically) time span of 6 seconds. It was assumed that the heat exchange between the sample and the surrounding cooling medium takes place through the convection with the appropriately selected value of the surface film conductance  $k$ . The numerical model of the process, and the values of such parameters as heating time and surface film conductance of water in particular, were selected based on experimental data concerning surface induction hardening of thin (4.5 mm of thickness) sheet made out of steel 55. This way of a construction of numerical model was used because neither the process analysis nor the modeling of induction heating constituted the aim of the work. On the basis of experimental data such values of heating time (the first stage of applying load) and surface film conductance (the second stage of applying load) were selected that the thickness of the hardened layer was approximately equal to its real value and the cooling time amounted to (established experimentally) 6 seconds.

Two parameters decide that a given layer has been hardened. One is austenitizing temperature and the other is the rate of cooling (amounting to at least  $60^{\circ}\text{C/s}$ ).

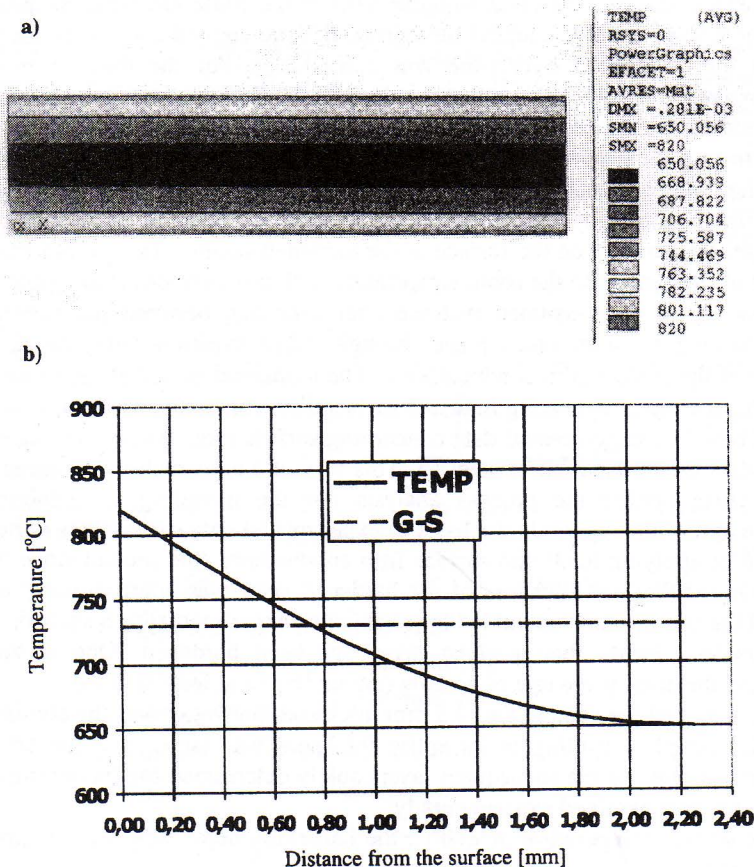
Due to the facts that the sample was 4.5 mm thick and that water was the cooling medium, the rate of the sample's cooling in its entire thickness was larger than the critical value. Therefore, temperatures of the subsequent layers solely determined the hardening depth. The following results were obtained experimentally:

- for the surface temperature of  $870^{\circ}\text{C}$ , the hardening depth amounts to approximately 1.2 mm, and
- for the surface temperature of  $820^{\circ}\text{C}$ , the hardening depth amounts to approximately 0.6 mm.

For the above data, heating time was determined for a selected mode of thermal load using trial-and-error method. Temperature distribution after 2 second of heating is presented in Figures 2 and 3.

As seen in Figures 2 and 3, the thickness of a hardened layer amounts to approximately 0.7 mm for the surface temperature of  $820^{\circ}\text{C}$  and to approximately 1.1 mm for the surface temperature of  $870^{\circ}\text{C}$ . These results well correspond to those obtained experimentally. As a result, the heating time in the numerical model (duration of the first stage of applying load) was accordingly assumed to be 2 seconds.

In the numerical model developed not only the variation of physical properties was taken into account but also phase transitions. Had the transition austenite/martensite not been taken into account, the results obtained would have been underrated. It was a formation of martensite, i.e. a structure whose density is higher up to 3%, that is responsible for the introduction of large internal stress. In order to take into account a phase transition taking place during material's cooling between  $M_s = 300^{\circ}\text{C}$  and  $M_f = 80^{\circ}\text{C}$  its properties, such as yield point and thermal expansion coefficient  $\alpha$ , were altered. A variation of  $\alpha$  was supposed to account for a change of the structure's volume during the phase transition. It is obvious that the discussed property changes for a hardened structure have to be different than for a not



**Figure 2.** Temperature distribution in a sample of steel after 2 seconds of heating at 820°C. a) a map of temperature distribution; b) temperature variation along sample's thickness.

hardened one. In order to account for a phase transition taking place before the second stage of loading, for those elements that are located in the determined depth of hardening the material properties were programmed in such a way that, in temperature range of 900 – 300°C, they were the same for both hardened and not hardened layers and differed in the temperature range that corresponded to austenite-martensite transition. A change of the yield point with temperature is based on literature data. As far the thermal expansion coefficient is concerned, its variation with temperature is programmed in such a way that the change of volume (including the formation of residual austenite) amounts to 1.5%.

According to the theory of elasticity, the change of volume may be expressed as:

$$\Delta V/V = \epsilon_x + \epsilon_y + \epsilon_z \quad (1)$$

where  $\epsilon_x$ ,  $\epsilon_y$  and  $\epsilon_z$  denote strain in three perpendicular directions.

If we want to express the volume change resulting from the phase transition by means of the change of thermal expansion coefficient, than we should use thermal strain in our calculations and, assuming that the thermal expansion coefficient is identical in all three directions (isotropic material) we can write that:



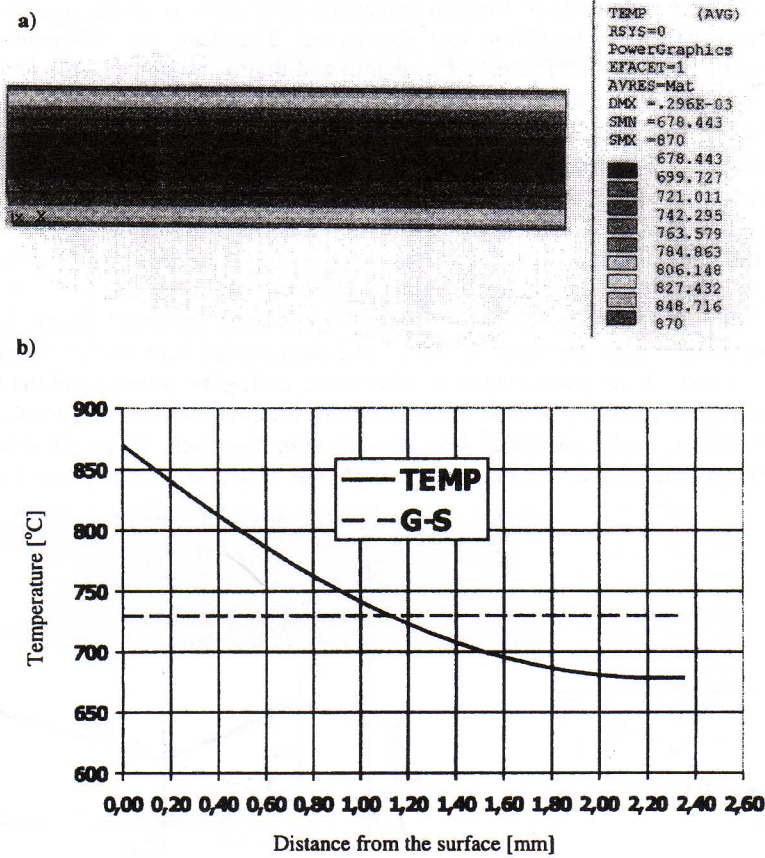


Figure 3. Temperature distribution in a sample of steel after 2 seconds of heating at 870°C. a) A map of temperature distribution; b) Temperature variation along sample's thickness.

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha_v \Delta T = \varepsilon_v \quad (2)$$

where  $\Delta T$  denotes temperature difference between the material and its environment. Combining (1) and (2) we obtain:

$$\Delta V/V = \alpha_v \Delta T = 0.015 \quad (3)$$

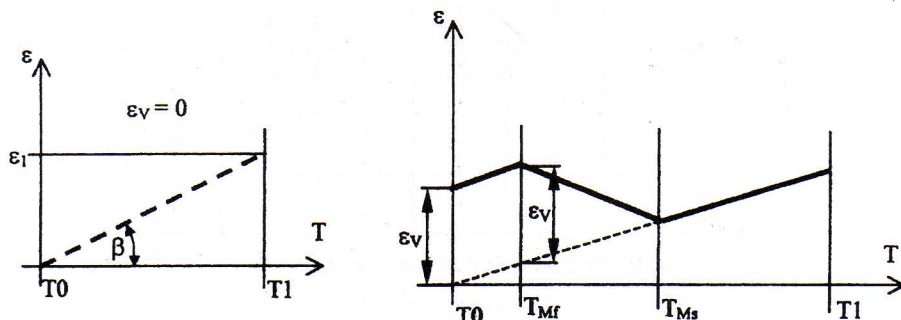
It is known that, for steel 55, the transition austenite-martensite begins at the temperature  $M_s = 300^\circ\text{C}$  and it is continued with the lowering temperature until the  $M_f$  point of about  $80^\circ\text{C}$  is reached. Therefore, the differences of the thermal expansion coefficient should begin at  $M_s$  and they should exhibit such a linear increase, that the value corresponding to the overall change of volume is reached at  $M_f$ . For the temperature  $M_f$  the value of  $\Delta T$  equals 80 deg and in that case we have:

$$\alpha_v = 0.015/3\Delta T = 6.25 \cdot 10^{-5} \quad (4)$$

In this way the magnitude of thermal expansion coefficient  $\alpha$  responsible for the volume change during the phase transition was determined. Therefore, the difference between the linear thermal expansion coefficient of austenite and that of martensite should increase from zero at  $T = M_s$  to the value of  $\alpha_v$  for  $T = M_f$  and then further to infinity for the reference temperature.

In practice, the increase of a difference between the  $\alpha$  coefficients of both phases is determined for the ambient temperature (the temperature to which a sample is cooled in the hardening process)  $T_a = 20^\circ\text{C}$ , with the assumption that the reference temperature equals  $T_{\text{ref}} = 0^\circ\text{C}$ . On the basis of the above calculations a linear thermal expansion coefficient for martensite was assumed (see Figure 1b).

The following reasoning should explain the procedure presented above. Let us assume that a given sample is heated from  $T_0$  to  $T_1$  and then cooled back to the initial temperature point. We assume that no phase transition takes place during the process and the material does not change its dimensions – in each direction strain is equal zero (see Figure 4, dashed line). In order to obtain, in the combined heating – cooling sequence, values of strain other than zero, the magnitude of thermal expansion coefficient must be changed (see Figure 4, solid line).



**Figure 4.** Plots of strain as a function of temperature – a scheme of strain formation as a result of a change of thermal expansion coefficient with temperature.  $\tan \beta = \alpha$  – thermal expansion coefficient,  $\epsilon_1 = \alpha (T_1 - T_0)$  – strain related to the increase of temperature,  $\epsilon_v$  – strain related to the volume change – according to the equation (2).

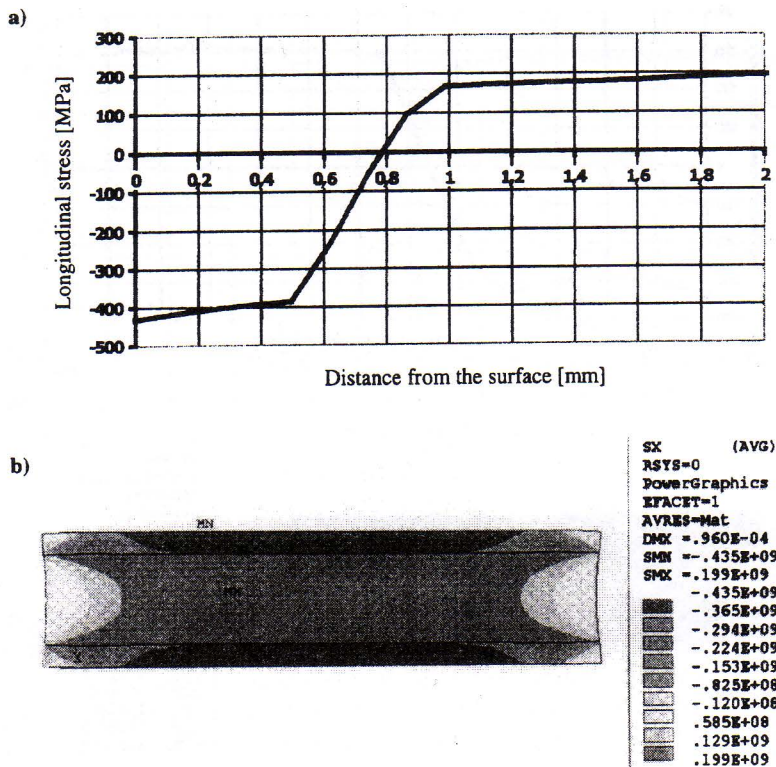
### 3. Results

The model developed was used to calculate the distribution of internal stress formed in the process of hardening steel for two different hardening temperatures, namely  $820^\circ\text{C}$  (see Figure 5) and  $870^\circ\text{C}$  (see Figure 6).

As seen in the Figure 5a, the character of internal stress distribution throughout the sample is similar to that of the observed, experimental results. What we observe in this case is an evident region of compressive stress (very advantageous from the viewpoint of both material's hardness and its fatigue strength), at the depth of approximately 0.8 mm converting to tensile stress that is characteristic for the core of the material. The simulated change of materials volume, due to a phase transition austenite-martensite, has resulted in the expected volume increase of the hardened material region, accompanied by an introduction of compressive stress in that region and a parallel formation of equivalent tensile stress in the core.

On the map of internal stress distribution (Figure 5b) the change of sample's form at its edges is also evident. In reality this change is, of course, a much smaller one; it has only been





**Figure 5.** Internal stress resulting from surface hardening at 820°C (sample 1).

a) a plot of internal stress value as a function of a distance from the sample's surface; b) a map of internal stress distribution in the sample's cross-section.

enlarged in the figure to a degree that it makes it noticeable. This change constitutes a typical, and often encountered, hardening strain which is relatively easy to predict and to correct in the case of such a simple element. The presence of this change in the discussed model is a confirmation of the correctness of the model's assumptions, and it allows us to take it for granted that a prediction and counteraction of such a strain will be also possible in the cases of more complicated forms.

Very similar results have been obtained for the hardening temperature of 870°C. They are presented in the Figure 6 below:

Also in this case we observe a region of compressive stress at the sample's surface, converting to tensile stress in its core. A detailed analysis of the plot presented in Figure 6a reveals the fact that the magnitude of tensile stress in the core exceeds the value of compressive stress in the hardened layer. This is very likely due to the higher thickness of the layer of a martensite structure, which introduces higher overall tensile stress in the core with the accompanying compressive stress relaxation in the layer itself.

It is also worth noting that, in a contrast to the sample heated to a lower temperature (sample #1), in the sample #2 presented in Figure 6b one can observe (along with a small deformation of the sample's edges) also a minor thickness change – the sample is slightly thinner in its middle than at its ends. Such a change of thickness is a result of stress fields

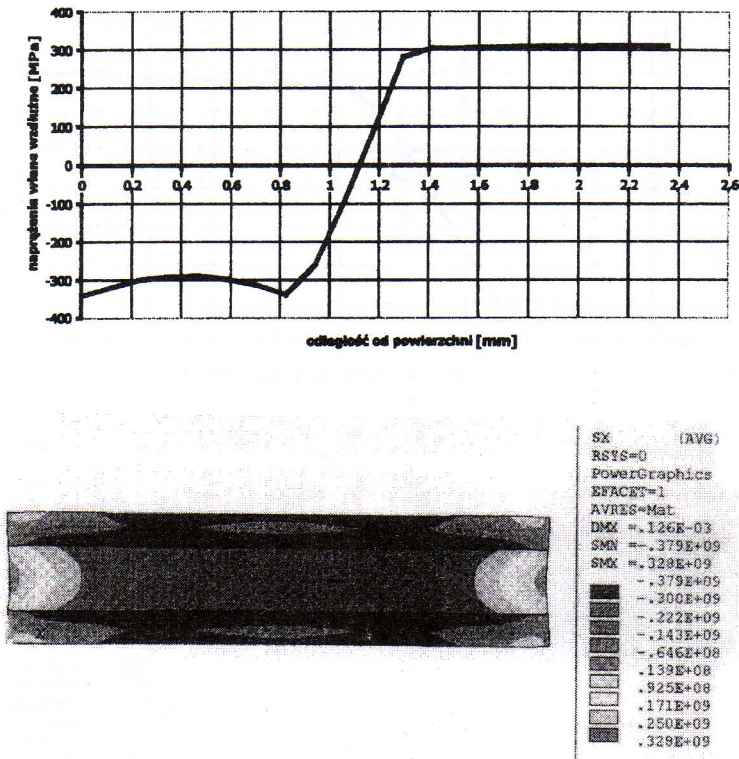


Figure 6. Internal stress resulting from surface hardening at 870°C (sample 2).

a) a plot of internal stress value as a function of a distance from the sample's surface; b) a map of internal stress distribution in the sample's cross-section.

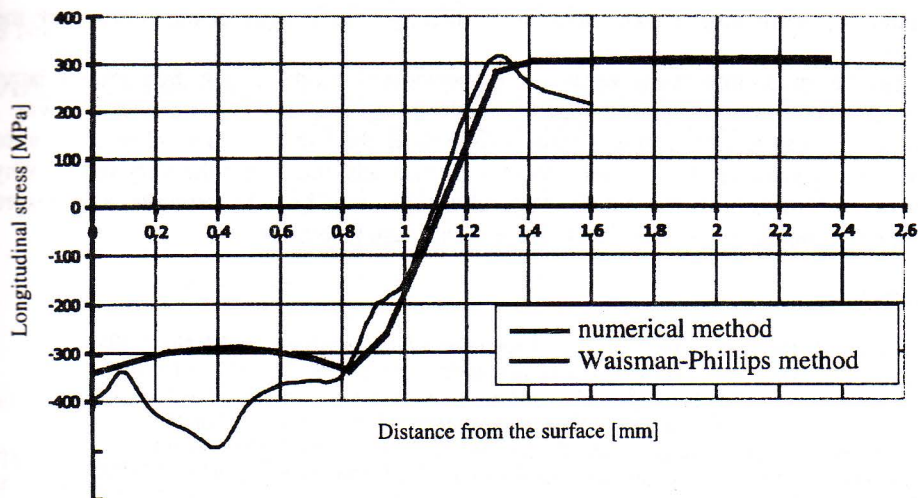
much higher than those present in the sample #1. This observation finds its confirmation in the fact that, in the sample #2, the region of low values of stress (of an order of magnitude of a dozen MPa), marked with light green colour, is very small. In contrast, such regions (marked with yellow colour) are a lot larger in the sample #1.

The most important property of any model is its relevance to reality. In order to test this, the results of numerical modeling should be verified with the experimental data. The results of numerical simulation obtained in the present work were compared with the results of internal stress measurements carried out using the Waismann-Phillips method. The comparison is presented in the Figure 7.

The results of internal stress distribution acquired with the Waismann-Phillips method exhibit a high degree of consistence with those obtained from the numerical simulation. The differences in the range of surface compressive stress may result either from measurement inaccuracies or (what is by far more probable) from imperfections of the model itself. Nevertheless, a general course of both curves is nearly identical, which constitutes a good argument as to the correctness of the assumptions and the performance of the numerical model.

A disadvantage of the presented model, which should be removed in it later versions, is the way compressive stress in the surface layer transforms into tensile stress in the core. It is





**Figure 7.** A comparison between internal stress distribution data obtained from model calculations and acquired experimentally.

seen in Figures 4b and 5b that the transition is of a discrete character – from compressive stress of 300-400 MPa to tensile stress of 200-300 MPa. This is due to the fact that the modelling software operates on a discrete model, which does not account for the continuity of the sample's structure. The situation presented in Figures 4b and 5b suggests a presence of extremely large shear stress in the border line between the layer and the core of the sample, far exceeding its shear strength. Such a situation is, certainly, impossible – in reality the transition from the structure of the layer and that of the core is somewhat “fluid” which is often observed during microscopic structure examination of hardened materials. Due to this fact the transition from compressive stress to tensile stress takes place in the region of a certain thickness and not instantaneously.

#### 4. Discussion

The results of internal stress obtained with the help of numerical model, based on the finite elements method, have shown that this method enables a relatively easy and quick determination of internal stress distribution in the volume of steel elements after their surface hardening. The existing numerical model still requires certain corrections. Nevertheless, a good agreement of the results computed with the help of this model with those obtained experimentally (by means of the destructive Waismann-Phillips method) allows us to judge on the correctness of both theoretical basis for the model and its construction.

The numerical model constructed above puts together a number of features that are not available for conventional methods. A determination of internal stress distribution with the use of this model is easy and relatively quick (much quicker than in the case of any other method). Also, a very important is the potential to quickly alter input load values which gives a possibility to study the effect of initial parameters of the hardening process on both the spectrum of internal stress and the strain of the hardened element. This possibility is of a particular importance, since it enables a selection of optimum parameters of the hardening process. It is especially important in the cases of elements having more complicated forms, for which numerical simulation may help to predict and to determine both the directions and

the values of hardening related strain and, therefore, to correct design details of those elements.

By far the most important feature of the discussed model is the fact that it is very economical. The results are gained quickly, in a low cost- and low labour-consuming way. An application of the model is much more economical than that of any other conventional method. This constitutes this model's most important advantage and the very reason why it has been constructed. In the future, this advantage may help to promote a widespread application of the model, as a substitute for presently used methods.

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