

THE CHAOTIC PHENOMENONS OF A SYSTEM WITH INERTIAL COUPLING

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The nonlinear response of two-degree-of-freedom vibratory beam-pendulum system in the neighbourhood internal and external resonances is investigated. The analysis was realised in the wide aspects of the influence of different kinds of nonlinearities, dampings and excitations. The equations of motion have been solved numerically. The present paper is a continuation of the author's previous works, where it was shown that in this type system one mode of vibration may excite or damp another one, and that except different kinds of periodic vibration there may also appear chaotic vibration. To prove the character of this vibration and to realise the analysis of transitions from periodic regular motion to quasi-periodic and chaotic, there have been constructed the bifurcation diagrams and time histories, phase plane portraits, power spectral densities, Poincaré maps and exponents of Lyapunov. These bifurcation diagrams show many sudden qualitative changes, that is, many bifurcations in the chaotic attractor as well as in the periodic orbits.

Keywords: beam-pendulum, nonlinearity, bifurcation, attractor.

1. Introduction

In this work the nonlinear dynamics of a two-degree-of-freedom beam-pendulum system is studied. Dynamically systems with elements of the mathematical or physical pendulum type have important applications. If the pendulum is suspended to the cantilever beam, in this system may occur the autoparametric excitation as a result of inertial coupling. In the systems of this type may occur the internal resonance of a parametric type. Fundamental is the influence of different types of nonlinearities on conditions of internal or external resonances.

The present paper is a continuation of the author previous works [1,2,3,4]. The equations of the autoparametric system have coupled nonlinearities and in this type systems can occurs except steady-state, also chaotic vibrations. It depends on various amplitudes of excitation, frequencies ratio and different parameters of the systems, for example various coefficients of the damping presented by author in [1].

A number of research have been discussed the complicated motion that may occur chaos in nonlinear mechanical systems with external excitation. A typical example of the governing equation is Duffing's equation with damping and harmonic or parametrically excited systems were presented by Moon [5]; Bajaj and Johnson [6]; Bajaj and Tousi [7]; Szemplińska-Stupnicka, [8,9]. Chaos in a nonlinear single-degree-of-freedom, parametrically excited system was observed by Szemplińska-Stupnicka at all [10]. There the excitation appears as a coefficient in the equation of motion (similarly as in the autoparametric systems).

Chaos for two degree-of-freedom autoparametric system was investigated by Hatwall at all [11]. Those authors used the harmonic balance method showed that for higher excitation levels, the response was found to be chaotic. This system was next investigated, using the averaging analysis, by Bajaj at all [12] and by Benerjee at all [13]. Those authors showed bifurcation analysis and Poincaré maps of the period and chaotic solution for different various detuning of frequency. There was assumed that system is weakly nonlinear. The analysis of transitions from periodic regular motion to chaotic motion for two degree-of-freedom systems were presented by Gonzales at all [14] or by Půst and Szöllös [15], and for autoparametric system by Mustafa and Ertas [16], by Tondl [17] or by Verhulst [18].

In the present paper the analysis was realised in the wide aspects of the influence of different kinds of nonlinearities, dampings and excitations. Except the Duffing type nonlinearity and the geometric nonlinearity as a result of the existence of the pendulum in the system, also the geometric nonlinearity as a result of large flexural deflections of the beam were taken into account. It was shown that in this type system one mode of vibrations may excite or damp another mode, and that except different kinds periodic vibrations may appear also chaotic vibration [1]. For characterising the previous discovered strange chaotic attractor, where the effect of small damping acting on the beam or on the pendulum for large flexural deflections of the beam, partially was demonstrated previous by author [2-4] time histories, phase plane portraits, power spectral densities, Poincaré maps and exponents of Lyapunov. In this work also the bifurcation diagrams for different damping parameters are constructed. When a bifurcation diagram is plotted, several phenomena can be observed: existence of a simple attractor with low period, or existence of a chaotic attractor, and various bifurcations [19]. All these phenomena have to be verified in the phase space. So in the present paper the time histories, phase plane portraits, power spectral densities, the Poincaré maps and exponents of Lyapunov also are constructed.

2. System description and equation of motion

The investigated system is shown in Fig.1. The system consists of a weightless beam of length l and stiffness EI . A body of mass m_1 is attached at the end of the beam. A pendulum of the length l_1 and mass m_2 hangs down from the body of mass m_1 . It is admitted that a linear viscous damping force acts upon the body m_1 and a proportional to angular velocity damping force applied in the hinge opposes the motion of the pendulum. The body of mass m_1 subjected to harmonic vertical excitation ($F=F_0\cos\eta t$). This system has two degrees of freedom: the vertical displacement y and the angle η . The relation between the horizontal displacement or vertical spring force and vertical displacements of the beam, also expression of the kinetic and the potential energy, the equations of motion were presented by author in [1]. As generalised coordinates are assumed the vertical displacement y of the body of mass m_1 measured from the equilibrium position and the angle ϕ of deflection of the body of mass m_2 measured from the vertical line.

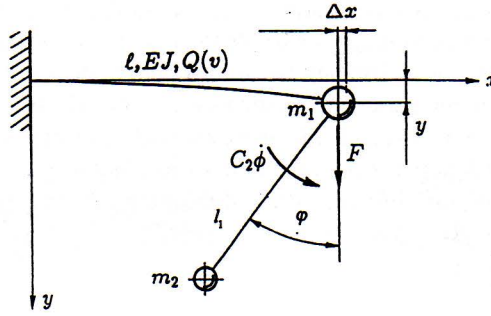


Figure 1 Schematic diagram of system

The equations of motion for the dimensionless time and for the dimensionless parameters are in form

$$\begin{aligned} \ddot{y}_1 = & \frac{1}{f_5} \left\{ \left[2\alpha_1 f_1 \cos \varphi - 4(1+b_1)\alpha_1^2 (y_1 + y_{1st}) \right] \dot{y}_1^2 + \right. \\ & - (b_1 + 1)f_4 + a_1 f_2 \dot{\varphi}^2 + a_1 \beta^2 (f_1 \sin \varphi + b_1 + 1) + \\ & \left. - b_1 f_3 \gamma_1 \dot{y}_1 + a_1 f_1 \gamma_2 \dot{\varphi} + A \cos \mu \tau \right\} \\ \ddot{\varphi} = & \frac{1}{a_1 f_5} \left\{ (1+d) \left[-2\alpha_1 f_3 \cos \varphi + 4\alpha_1^2 f_1 (y_1 + y_{1st}) \right] \dot{y}_1^2 + \right. \\ & + (1+d) \left[f_1 f_4 - a_1 \beta^2 (f_1 + f_3 \cos \varphi) - a_1 f_3 \gamma_2 \dot{\varphi} \right] + \\ & \left. - a_1 f_1 f_2 \dot{\varphi}^2 + b_1 f_1 f_3 \gamma_1 \dot{y}_1 - f_1 A \cos \mu \tau \right\} \end{aligned} \quad (1)$$

where

$$\begin{aligned} f_1 &= 2\alpha_1 (y_1 + y_{1st}) \cos \varphi - \sin \varphi \\ f_2 &= \cos \varphi + 2\alpha_1 (y_1 + y_{1st}) \sin \varphi \\ f_3 &= 1 + 4\alpha_1^2 (y_1 + y_{1st})^2 \\ f_4 &= (y_1 + y_{st}) + b_2 (y_1 + y_{st})^3 \\ f_5 &= (b_1 + 1)f_3 - f_1^2 \end{aligned} \quad (2)$$

and

$$\begin{aligned} y_1 &= \frac{y}{l}, \quad y_{1st} = \frac{y_{st}}{l}, \quad a_1 = \frac{l_1}{l}, \quad b_1 = \frac{m_1}{m_2}, \quad \beta = \frac{\omega_2}{\omega_1}, \quad \omega_1 = \frac{k_1}{m_1 + m_2}, \\ \omega_2 &= \frac{g}{l_1}, \quad \mu = \frac{\eta}{\omega_1}, \quad \gamma_1 = \frac{c_1}{m_1 \omega_1}, \quad \gamma_2 = \frac{c_2}{m_2 l^2 \omega_1}, \quad A = \frac{F_0}{m_2 l \omega_1^2}. \end{aligned} \quad (3)$$

3. Numerical results

Equations (1) were solved numerically by using the Runge-Kutta procedure. Near the internal and external resonances depending on a selection of physical system parameters the amplitudes of vibrations of both coupled bodies may be related differently, motions: y_1 and pendulum are periodic or quasiperiodic vibrations, but sometimes the motions of the beam and pendulum are chaotic. For characterising an irregular chaotic response forms

a transition zone between one and another type of regular steady resonant motion, in the present paper the bifurcation diagrams for different damping parameters are constructed. Exemplary results, for small damping put on pendulum, near internal and external resonances (near principal autoparametric resonance for $\beta=0.51$ and near external resonance for $\mu=1$), are presented in Figs 2–5, where are showed displacements and velocities of the beam end of the pendulum versus amplitude of excitation (scale A is compact). These diagrams are presented for different values of frequency β : in Fig.2 for $\beta=0.45$, in Fig.3 for $\beta=0.5$, in Fig.4 for $\beta=0.51$ and in Fig.5 for $\beta=0.52$.

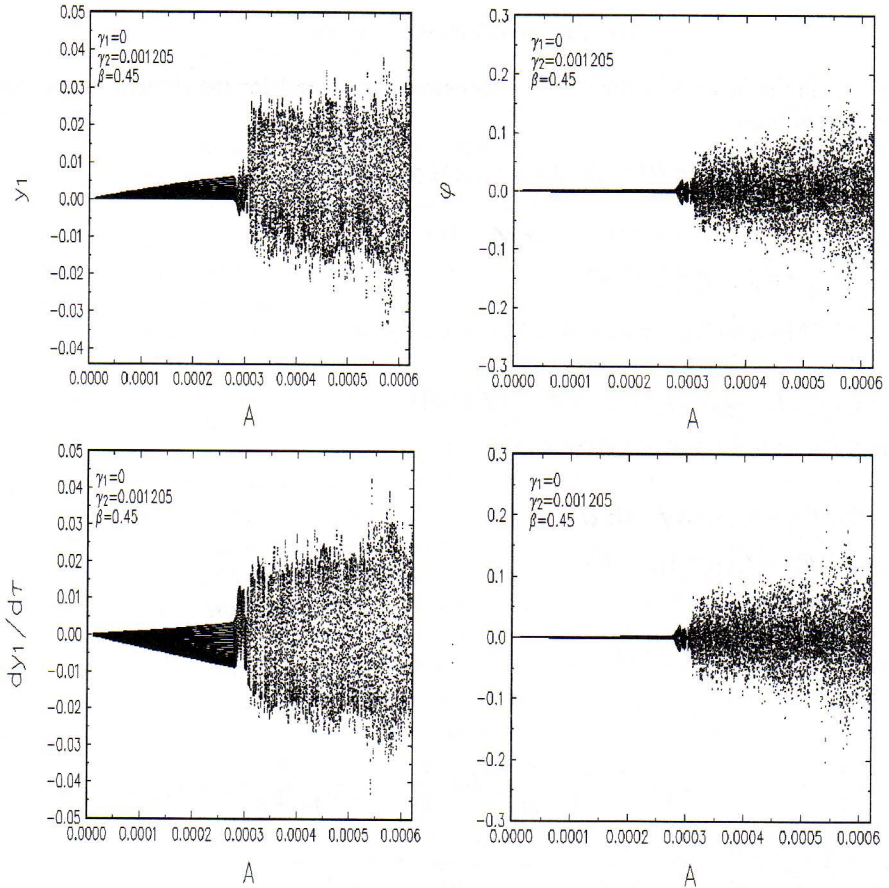


Figure 2 Bifurcation diagram for y and ϕ for $\beta=0.45$ and: $b_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $m_1=0.99$

As can be seen from diagrams presented in Fig.2 in this case ($\beta=0.45$) for small value of the excitation amplitude A ($A < 0.00028$) both motions beam and pendulum are periodic, but for greater value of parameter A character these motions may be irregular.

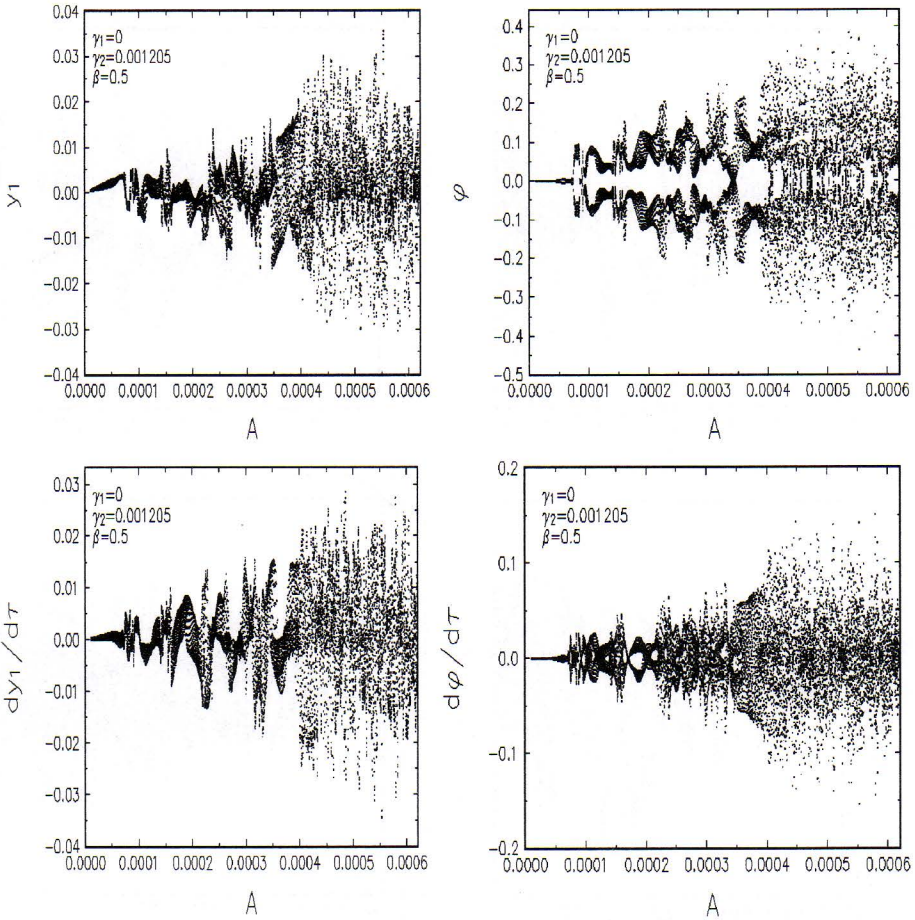


Figure 3 Bifurcation diagram for y and ϕ for $\beta=0.5$ and: $b_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $\mu_1=0.99$

As can be seen from diagram presented in Fig.3 for parameter $\beta=0.5$ periodic motions are for value of amplitude of excited $A < 0.00005$, and for greater value of parameter A the motion of the beam and of the pendulum have different character: may be periodic, quasiperiodic or irregular.

As can be seen from Fig.4 ($\beta=0.51$) and from Fig.5 ($\beta=0.52$), in this case for greater value of frequency parameter the range of value of the parameter A when the motions of the beam and pendulum are periodic is longer. ($A < 0.0001$ for $\beta=0.51$ and $A < 0.00017$ for $\beta=0.52$) and for greater value of parameter A the motion of the beam and of the pendulum have different character: may be periodic, quasiperiodic or irregular.

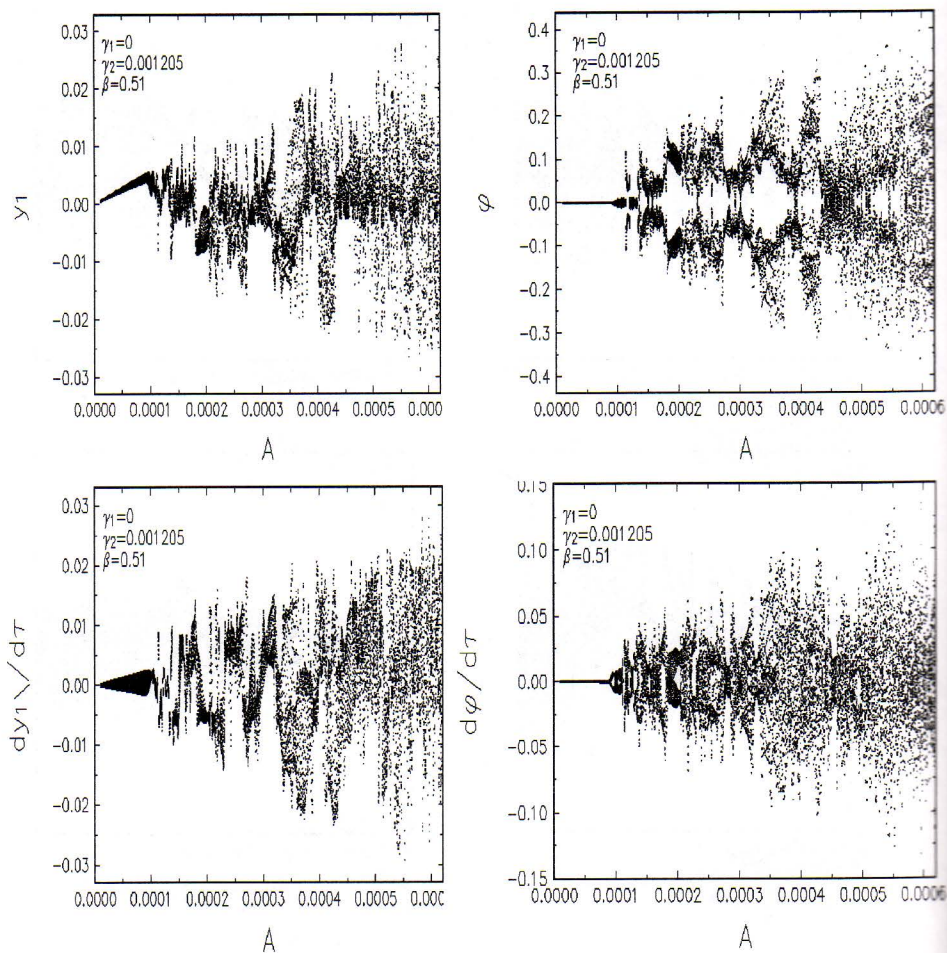


Figure 4 Bifurcation diagram for y and φ for $\beta=0.51$ and: $b_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $\mu_1=0.99$

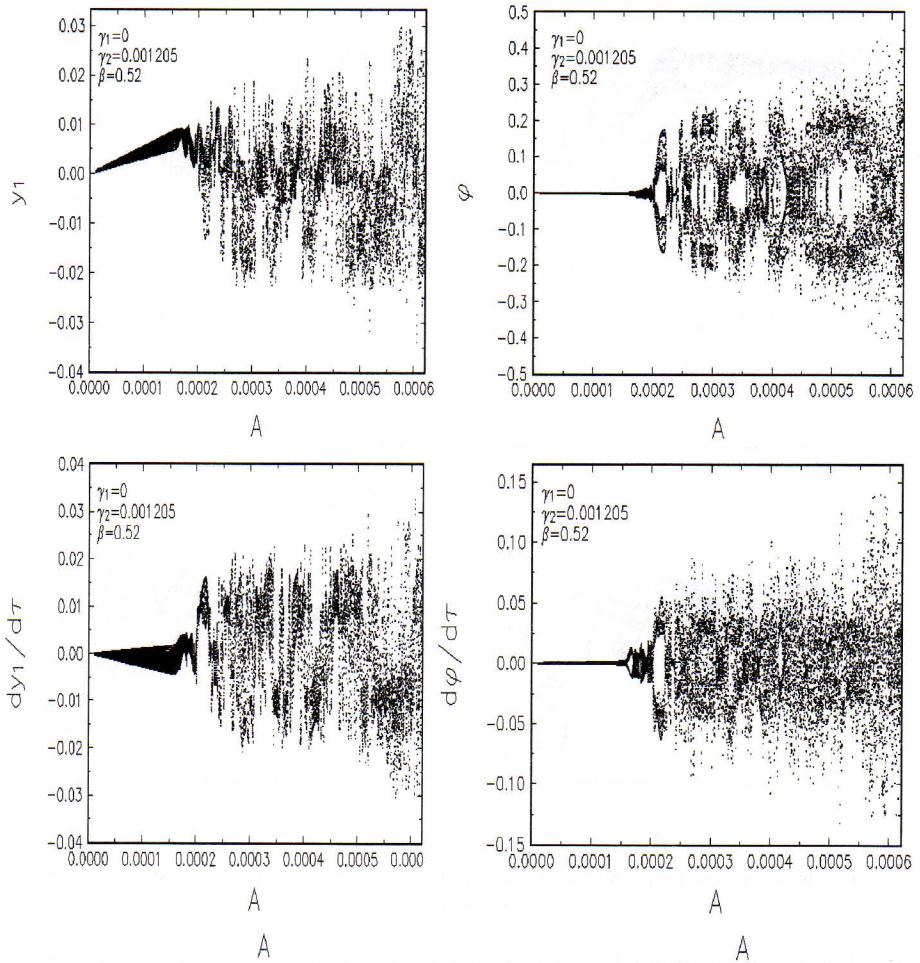


Figure 5 Bifurcation diagram y for different values of $\beta=0.52$ and: $\beta_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $\mu_1=0.99$

As it can be seen from Figs 2–5 even small change of the frequency ratio (parameter β) gives different character of the beam and pendulum motions for the some values of amplitude excitation (parameter A). As can be seen from these bifurcation diagrams several phenomena can be observed: existence of a simple attractor with low period, or existence of a chaotic attractor, and various bifurcations. These phenomena can be observed sometimes better for displacement, sometimes for velocities, so next diagrams are presented for both in tension scale for small damping put on the beam, on the pendulum or together. In Fig.6 are presented these bifurcation diagrams for small damping put on the beam.

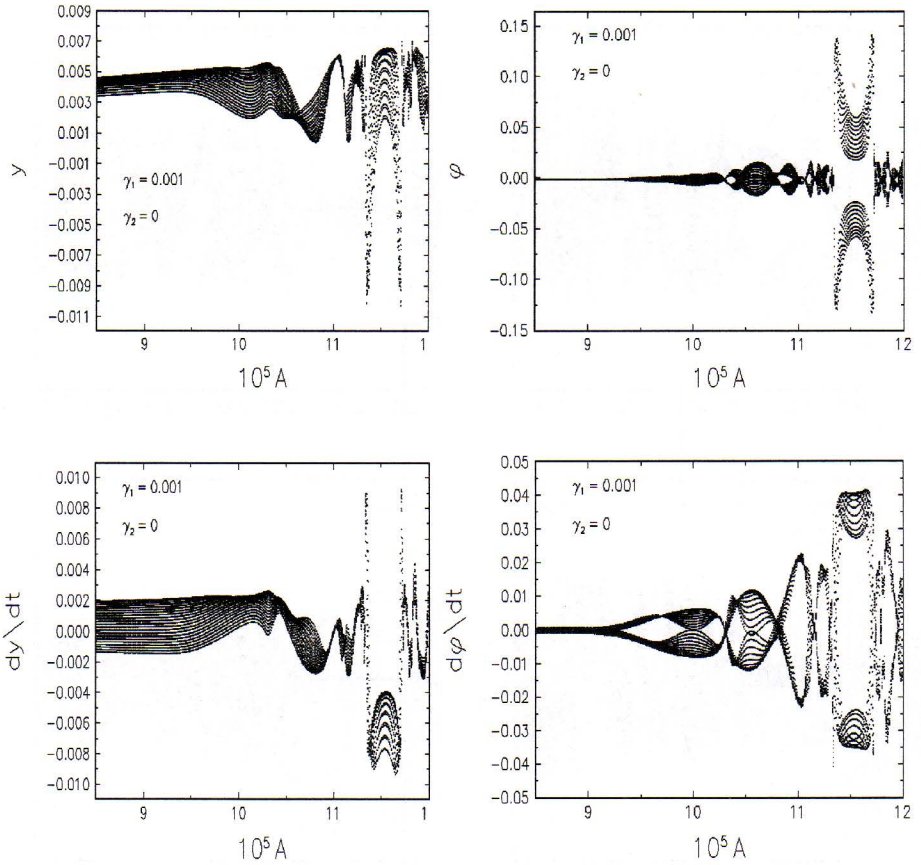


Figure 6 Bifurcation diagrams y_1 and ϕ for damping put on the beam ($\beta_1=0.51$; $b_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $\mu_1=0.99$)

The bifurcation diagrams for small damping put on the pendulum are presented in Fig. 7. Next, the some bifurcation diagrams for small damping put on the beam and on the pendulum together are presented in Fig. 8.

Even small change the damping parameter of the system gives different character of the beam and pendulum motion. As can be seen from these bifurcation diagrams, several phenomena can be observed existence of a simple or chaotic attractor, and various bifurcations. All these phenomena have to be verified in the phase space. Next than the time histories, phase plane portraits (Poincaré maps), power spectral densities (their fast Fourier transform -FFT), and the exponents of Lyapunov are constructed. This descriptors are available to observe chaos, and to better understand it.

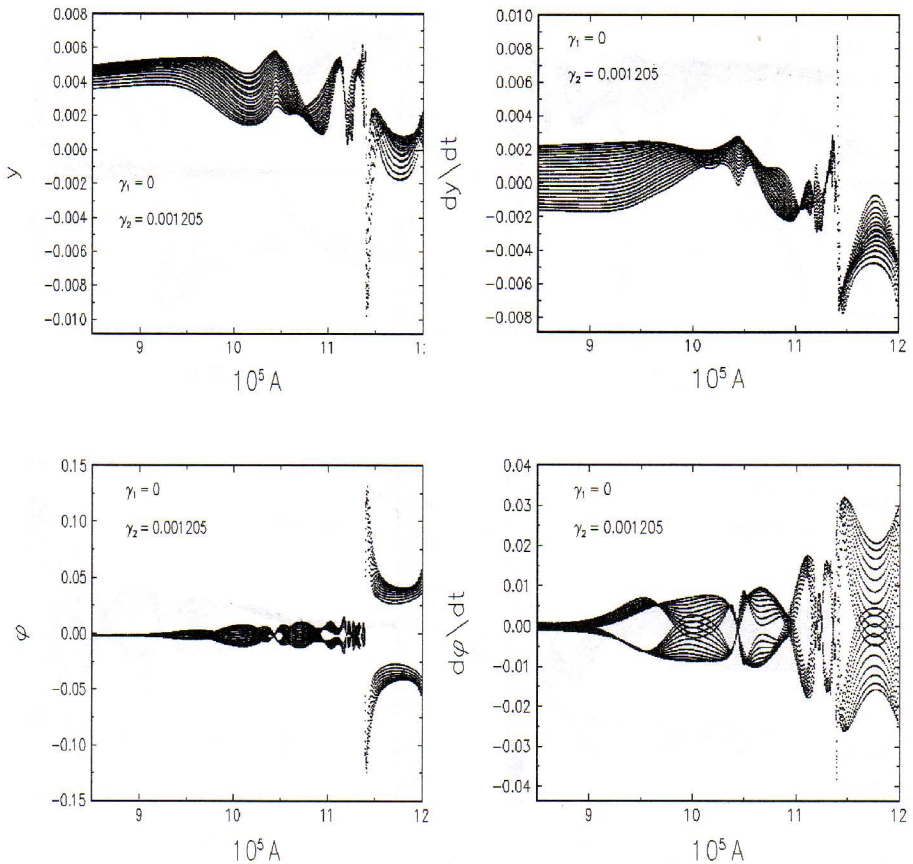


Figure 7 Bifurcation diagrams y_1 and ϕ for damping put on the pendulum ($\beta=0.51$; $b_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $\mu_1=0.99$)

Exemplary results chaotic motions are presented in Fig. 9 (time histories, power spectral densities (FFT), Poincaré maps and the maximum exponents of Lyapunov corresponding to coordinate y_1 as well as to coordinate ϕ).

As can be seen from Fig. 9 this response is chaotic. We see that in this case the motion looks like irregular, the Poincaré maps trace the *strange attractors*, the frequency spectrums are continuous and the largest exponents of Lyapunov are positive.

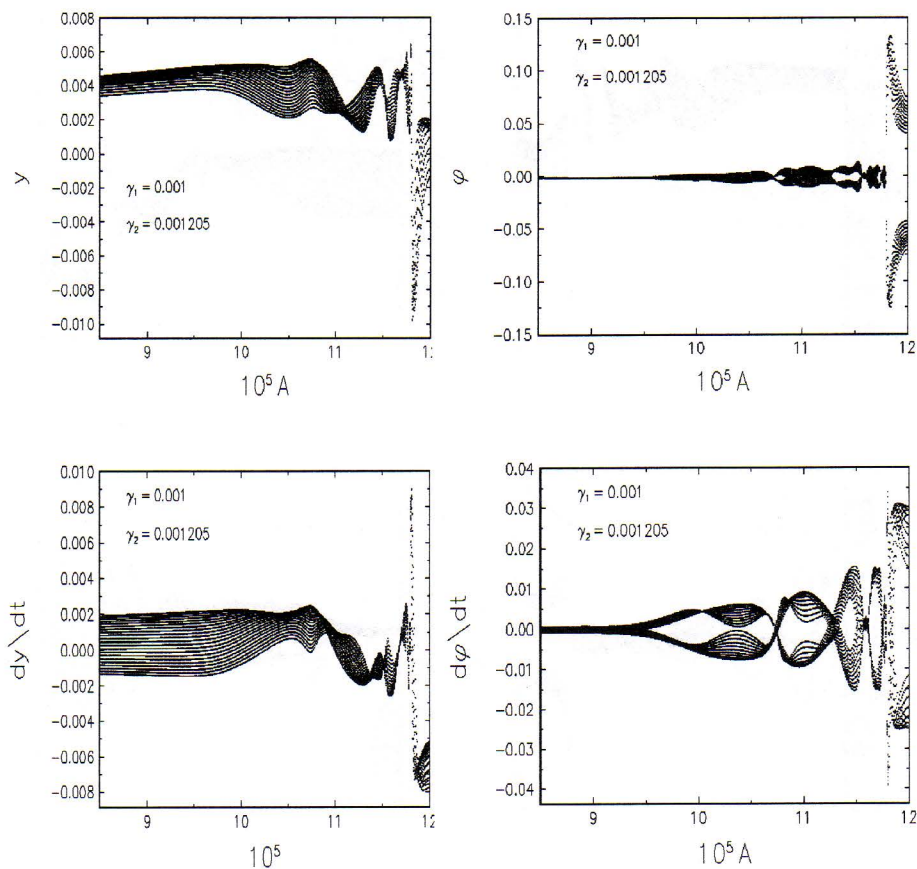


Figure 8 Bifurcation diagrams y_1 and ϕ for damping put on the beam and on the pendulum together ($\beta=0.51$; $b_1=0.2$; $a_1=0.2$; $\alpha_1=0.5955$; $b_2=1$; $\mu_1=0.99$)

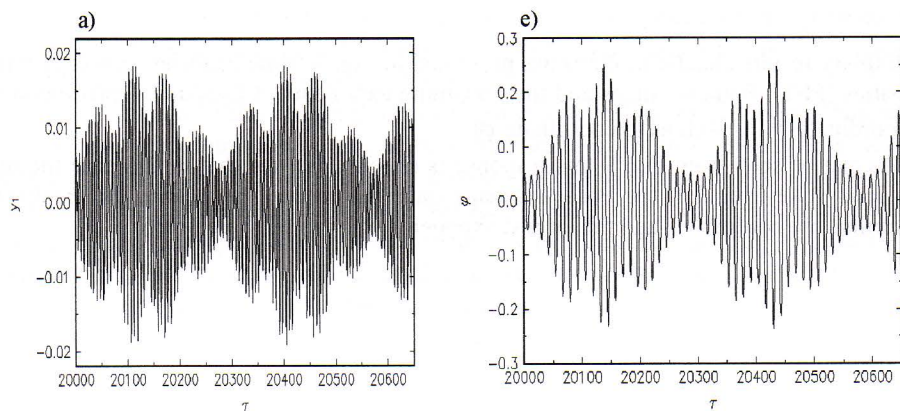


Figure 9 See description on the next page

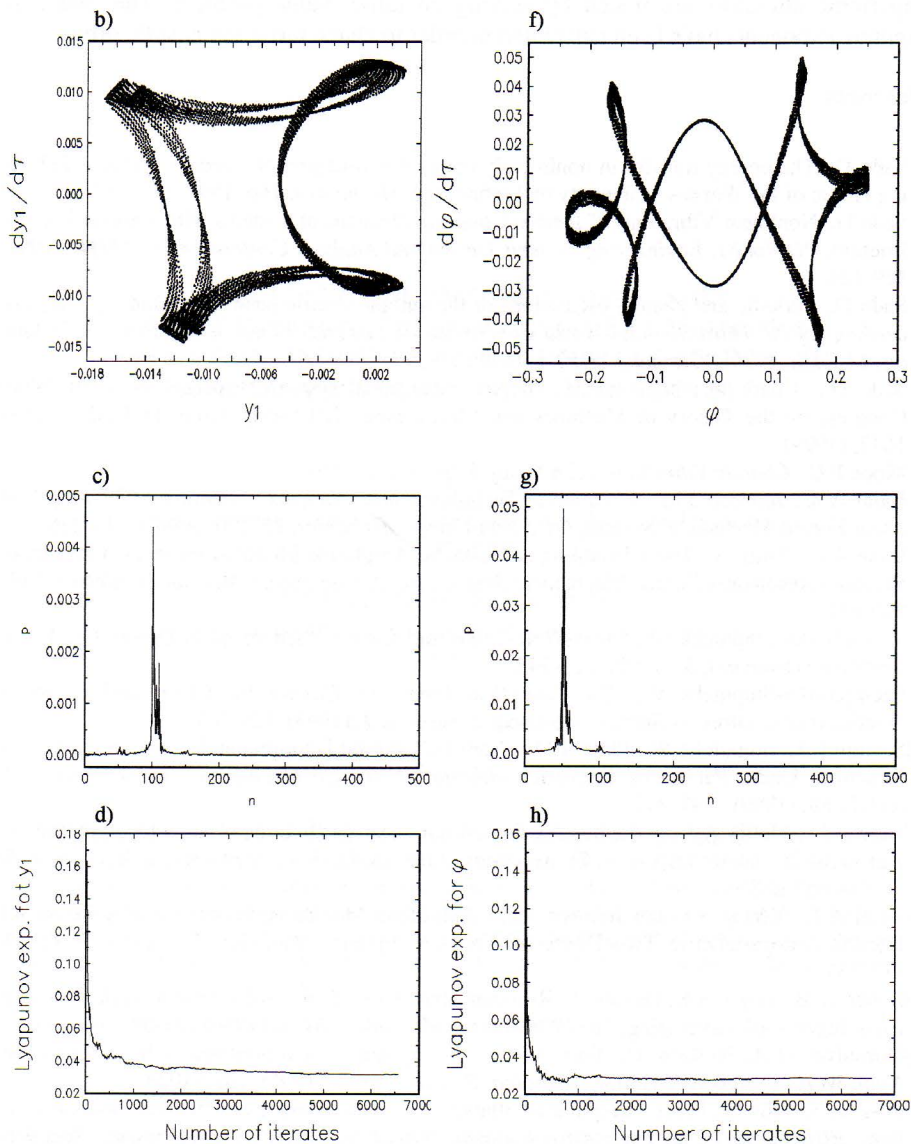


Figure 9 Time history, power spectral density (FFT), Poincaré map and maximum Lyapunov exponents corresponding to coordinate y_1 (a – d) and to coordinate ϕ (e – h) for: $\beta=0.52$; $\alpha_1=0.5955$, $a_1=0.2$; $b_1=0.2$; $b_2=1$; $\gamma_1=1$; $\gamma_2=0.001205$; $A=0.0006$

4. Conclusion

Influence of parameters on the behaviour of the autoparametric beam-pendulum system near the internal and external resonances frequencies is very interesting and important. In autoparametric system the energy is transferred from one degree of freedom to the other. Depending on a selection of physical system parameters the amplitudes of vibrations of both coupled bodies may be related differently. It was shown that except different kinds periodic vibrations might appear also different kinds irregular vibrations.

Nonperiodic attractors are traced by solving an initial value problem. The maximum Lyapunov exponents have been calculated in order to characterise the chaotic orbits.

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