# Effect of Geometrical Orientation of Three-Lobe Slide Bearings on Amplitude of Resonant Vibration of an Unsymmetric Supported Rotor 

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#### Abstract

In the paper the effect of geometric orientation of bearing shells on damping of forced vibration of journals of an asymmetric rotor has been analysed. Considerable influence of mutual position of the shells, has been confirmed as far as decrement of the vibration amplitude was concerned. The effect of asymmetry oil clearances and journal lengths has been taken into account. The results are shown in appropriate graphs.


Keywords: Rotor, resonant, clearance.

## 1. Introduction

The undesirable damping vibrations, means self-exited or resonant vibrations in the slide bearing systems are a very important problem, which directly connect with durability and reliability. In general, to eliminate arising vibrations in the system suitable methods can be used and divided into active and passive.

The active methods usually demand an enlargement of other elements in the slide bearing systems, which directly correspond to cost increase. However, the passive methods only demand a change in some properties in the used system, means between the journal - oil film - bush bearing. One of these changes is to apply a differently geometrically shaped bush bearing. It occurred, that a different than cylindrical bush bearing shape can ensure a better static work regime for the supported rotor as well as to analyse the self-excited vibrations and resonant vibrations. The research's work, where the above problems were mentioned and considered, can be presented for example by Osiński and Starczewski [1] and Starczewski [2]. In the research of Starczewski [3] as well as Przybyłowicz and Starczewski [4], the interesting aspect of the limiting arising amplitude vibrations for slide bearings where bush bearing has a non-linear contour throughout taking advantage of the geometrical orientation in the assumed co-ordinate system has been
presented. This precisely concerns the mutual revolutions of the bush bearing. It turns out that the vibration amplitudes can be significantly limited in bearings within these straightforward methods. The author's intention of this article is as follows : to test the possibilities of limiting vibration amplitudes of journals in the unsymmetrical rotor in inertial input (unbalance according to the rotation axes) by their changing positions (geometrical orientation) depending on the asymmetric degree and various structural parameters (bearing slackness, bearing length) of the system.

## 2. Model of the system and the bearing support

The model of the rigid rotor supported on the short slide bearings (perycycloidal contour) has been taken into consideration. The asymmetry of the system come from the unsymmetrical position of the external load Q . The short slide bearings system is characterized by neglecting during integrations Reynold's equation of the circumferential flow and concentrating on the pressure distribution along the bearing length. This model perfectly describes the actual slide bearings properties for the short slide bearings (length - diameter ration $L / D$ is less than $1 / 2$ ).

The accurate description of the pressure distribution for cylindrical bearings for dynamical needs, means the whole motion components of the journal centre (own rotation with speed rotation $\omega$, circumference plane motion with velocity $\dot{\alpha}$, radial motion connected to the squeeze effect with velocity $\dot{\beta}$ ) are taken into account. The precise description can be found in Kirk's and Gunter's works [5]. The flow of the lubricant factor is isometric (not depending on temperature and pressure). Even for such a simple support bearing model the Reynold's equation is impossible to solve. To describe the oil gap in the pericycloidal slide bearings with multiplication factors 3, the author used the geometrical contour approximation method presented by Kaniewski and Stasiak [6]. This method concerned the phenomenon of the statics in the pericycloidal bearing for the pressure distribution and the components of the oil hydrodynamic uplift forces in the state conditions. These conditions specify the relations between the oil film pressure and the components of the hydrodynamic uplift forces only from rotation speed $\omega$ of the journal centre in the situation when the journal is positioned on the so-called static equilibrium journal centre position curve. Generally, Kaniewski's and Stasiak's method consisted of the pericycloidal contour for individual bearing shell sections which had been replaced with the assembling of a few circular contours with definite approximation factors. This method, by the author of this article, has been adapted for dynamical needs. The assumed model of the system and the journal bearing has been presented below.

Figure 1 shows:
$L_{1}, L_{2}$ - bearing length
$R_{0 k}$ (for $k=1,2$ ) - short radius in the pericycloidal contour
$R_{z k}$ (for $k=1,2$ ) - journals radius " 1 " and " 2 "
$e_{k}($ for $k=1,2)$ - journals eccentricity " 1 ' and " 2 "
$\mathrm{O}_{1}$ - journals centre for bearing shell " 1 " and " 2 "

O - bearings centre " 1 " and " 2 "
$\alpha_{k}($ for $k=1,2)$ - angle of the line between the journal centre and the centre of the bush bearing for bearings " 1 " and " 2 "
$\varphi$ - current angle
$\delta_{k}$ (for $k=1,2$ ) - orientation angle of the bearing shell " 1 " and "2"
$r$ - unbalance radius
$a, b$ - coordinates defining the point of application of the external loading $Q$ in the Cartesian co-ordinates system $x, y, z$


Figure 1 The model of the system and journal bearing

Analytical dependence of the pressure distribution in the each sections of the slide bearing can be seen in the literature [1].

## 3. Equation of the rotor motion

On the basis of the analytical equations, which describe pressure distribution in each sections of the slide bearing, the hydrodynamical uplift forces in the directions x and $y$ have been defined. Through the integration of pressure distribution in each section for pericycloidal approximation contour of the bearing shell, the components of the hydrodynamical uplift forces, different conditions for the wedge effect and different conditions for the squeeze effect allow equations of motion to be written down.

The system of equations describing the journal motions in the bearings can be written down on the basis of Kurnik's and Starczewski's work [7] as follows:

$$
\begin{align*}
\ddot{x}_{1} & =\frac{I_{2} a}{I_{1} l} \omega\left(\dot{y}_{2}-\dot{y}_{1}\right)+\left(\frac{1}{m}+\frac{a^{2}}{I_{1}}\right) P x_{1}+\left(\frac{1}{m}-\frac{a b}{I_{1}}\right) P x_{2}-r \omega^{2} \sin \omega t  \tag{1}\\
\ddot{y}_{1} & =\frac{I_{2} a}{I_{1} l} \omega\left(\dot{x}_{1}-\dot{x}_{2}\right)+\left(\frac{1}{m}+\frac{a^{2}}{I_{1}}\right) P y_{1}+\left(\frac{1}{m}-\frac{a b}{I_{1}}\right) P y_{2}-\frac{Q}{m}-r \omega^{2} \cos \omega t \\
\ddot{x}_{2} & =\frac{I_{2} a}{I_{1} l} \omega\left(\dot{x}_{1}-\dot{x}_{2}\right)+\left(\frac{1}{m}+\frac{a^{2}}{I_{1}}\right) P y_{1}+\left(\frac{1}{m}-\frac{a b}{I_{1}}\right) P y_{2}-\frac{Q}{m}-r \omega^{2} \cos \omega t
\end{align*}
$$

$$
\ddot{y}_{2}=\frac{I_{2} b}{I_{1} l} \omega\left(\dot{x}_{2}-\dot{x}_{1}\right)+\left(\frac{1}{m}+\frac{a b}{I_{1}}\right) P y_{1}+\left(\frac{1}{m}-\frac{b^{2}}{I_{1}}\right) P y_{2}-\frac{Q}{m}-r \omega^{2} \cos \omega t
$$

where:
$l$ - the rotor length
$m$ - rotor mass
$I_{1}, I_{2}$ - moments of inertia adequately to $x, y$ axes
$\dot{x}_{1}, \dot{y}_{1}, \dot{x}_{2}, \dot{y}_{2}$ - velocities of the journal in the $x, y$ directions for the bearings " 1 " and " 2 "

Individual components of the hydrodynamical uplift forces for bearings "1" and " 2 " with $k=1,2$ can be written as:

$$
\begin{align*}
& P_{x k}=\sum_{i=1}^{3}\left[\frac { \mu _ { k } L _ { k } ^ { 3 } } { 2 \varepsilon _ { k } \delta _ { k } } \left[\int_{\varphi_{p i}+\delta_{k}}^{\varphi_{k i}+\delta_{k}} \frac{\left(\left(\omega-2 \dot{\alpha}_{k}\right) \beta_{k} \sin \left(\varphi-\alpha_{k}\right)+\omega \lambda^{*} K_{n} \sin (A-\varphi)\right) \sin \varphi d \varphi}{\left(1+\beta_{k} \cos \left(\varphi-\alpha_{k}\right)+\lambda^{*} K_{n}(1-\cos (A-\varphi))\right)^{3}}+\right.\right. \\
& \left.\left.\quad-\int_{\varphi_{w p i}+\delta_{k}}^{\varphi_{w k i}+\delta_{k}} \frac{2 \dot{\beta}_{k} \cos \left(\varphi-\alpha_{k}\right) \sin \varphi d \varphi}{\left(1+\beta_{k} \cos \left(\varphi-\alpha_{k}\right)+\lambda^{*} K_{n}(1-\cos (A-\varphi))\right)^{3}}\right]\right]  \tag{2}\\
& P_{y k}=\sum_{i=1}^{3}\left[\frac { \mu _ { k } L _ { k } ^ { 3 } } { 2 \varepsilon _ { k } \delta _ { k } } \left[\int_{\varphi_{p i}+\delta_{k}}^{\varphi_{k i}+\delta_{k}} \frac{\left(\left(\omega-2 \dot{\alpha}_{k}\right) \beta_{k} \sin \left(\varphi-\alpha_{k}\right)+\omega \lambda^{*} K_{n} \sin (A-\varphi)\right) \cos \varphi d \varphi}{\left(1+\beta_{k} \cos \left(\varphi-\alpha_{k}\right)+\lambda^{*} K_{n}(1-\cos (A-\varphi))\right)^{3}}+\right.\right.  \tag{3}\\
& \left.\left.-\int_{\varphi_{w k i}+\delta_{k}} \frac{2 \dot{\beta}_{k} \cos \left(\varphi-\alpha_{k}\right) \cos \varphi d \varphi}{\left(1+\beta_{k} \cos \left(\varphi-\alpha_{k}\right)+\lambda^{*} K_{n}(1-\cos (A-\varphi))\right)^{3}}\right]\right]
\end{align*}
$$

where:
$P_{x k}, P_{y k}$ - the hydrodynamical uplift forces in the direction $x, y$ for the bearings "1" and "2"
$\mu_{k}$ - absolute viscosity of the lubricant factor for bearings " 1 " and " 2 "
$\varepsilon_{k}$ - absolute clearance in the bearings " 1 " and " 2 "
$\lambda^{*}$ relative eccentricity of the pericycloid
$K_{n}$ - pericycloidal approximation coefficient for bearings " 1 " and " 2 "
$\beta_{k}$ - relative eccentricity for bearings " 1 " and "2"; $\beta_{k}=e_{k} / \varepsilon_{k}$
$\varphi_{k i}, \varphi_{p i}$ - the integration boundaries for the bearings " 1 " and " 2 " - the wedge effect
$\varphi_{w k i}, \varphi_{w p i}$ - the integration boundaries for the bearings " 1 " and " 2 " - the squeeze effect
$\dot{\alpha}_{k}, \dot{\beta}_{k}$ - tangential and radius velocity in the bearings " 1 " and " 2 "
$A$ - factor which depends on the bearing sections for $i=1 \rightarrow A=\frac{\pi}{3} ; i=2 \rightarrow$ $A=\pi$.
$i=3 \rightarrow A=\frac{5 \cdot \pi}{3}$.
Because the examination of the journal motion centre takes place in the Cartesian co-ordinates system $x, y$ (the plane motion), the expressions for $\dot{\alpha}_{k}, \dot{\beta}_{k}, \alpha_{k}, \beta_{k}$ have to be described as follow form:

$$
\begin{align*}
\beta_{k} & =\frac{\sqrt{x_{k}^{2}+y_{k}^{2}}}{\varepsilon_{k}} \quad \dot{\beta}_{k}=\frac{x_{k} \dot{x}_{k}+y_{k} \dot{y}_{k}}{\beta_{k} \varepsilon_{k}^{2}} \quad \dot{\alpha}_{k}=\frac{x_{k} \dot{y}_{k}+y_{k} \dot{x}_{k}}{\beta_{k}^{2} \varepsilon_{k}^{2}} \\
\cos \alpha_{k} & =-\frac{x_{k}}{\beta_{k} \varepsilon_{k}} \quad \sin \alpha_{k}=-\frac{y_{k}}{\beta_{k} \varepsilon_{k}} \tag{4}
\end{align*}
$$

where:

$$
\begin{aligned}
& k=1 \text { for the bearing " } 1 \text { ", } \\
& k=2 \text { for the bearing " } 2 \text { ". }
\end{aligned}
$$

and taken into account in the equations (2) and (3).

## 4. Numerical results

The obtained system of the equations is strongly out of line because of the displacements and velocities and because they are coupled to each other as well, therefore it is impossible to answer them in an analytical way. That is why for these analyses we are obliged to use a digit simulation method. To solve the system of the equations the Mathematica 4.2 software package was used. The numerical results are presented below on the Figures 2-4.

## 5. Concluding remarks

There exists strong influence of geometric orientation on amplitude of forced vibration of the rotor (considerable chop of the amplitude).

For a given asymmetry of the rotor the minimum of vibration amplitude is found at a suitable angular position of the bearing shells.

The minimum of the amplitude is also observed for certain combinations of such parameters as bearing gaps and journal lengths.

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Figure 2 Effect of angular position of the bearing shell $\delta_{k}$ on amplitude of forced vibration A1 (bearing 1) and A2 (bearing 2) for various asymmetries of the rotor


Figure 3 Effect of angular position of the bearing shell $\delta_{k}$ on amplitude of forced vibration A1 (bearing 1) and A2 (bearing 2) for various bearing gaps


Figure 4 Effect of angular position of the bearing shell $\delta_{k}$ on amplitude of forced vibration A1 (bearing 1) and A2 (bearing 2) for various journal lengths
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