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Elastodynamics of Inclined Loads in a Micropolar Cubic Crystal

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The analytic expressions for the displacement components, microrotation and stresses at any point in an infinite micropolar cubic crystal as a result of inclined load of arbitrary orientation have been obtained. The inclined load is assumed to be a linear combination of a normal load and a tangential load. The eigenvalue approach using Laplace and Fourier Transforms has been employed and the transforms has been inverted by using a numerical technique. The numerical results are illustrated graphically for a particular material.

 $Keywords\colon$ Micropolar cubic crystal, inclined load, eigenvalue, Laplace and Fourier transform.

1. Introduction

The classical theory of elasticity does not explain certain discrepancies that occur in the case of problems involving elastic vibrations of high frequency and short wave length, that is, vibrations due to the generation of ultrasonic waves. The reason lies in the microstructure of the material which exerts a special influence at high frequencies and short wave lengths.

An attempt was made to eliminate these discrepancies by suggesting that the transmission of interaction between two particles of a body through an elementary area lying within the material was affected not solely by the action of a force vector but also by a moment (couple) vector. This led to the existence of couple stress in elasticity. Polycrystalline materials, materials with fibrous or coarse grain structure come in this category. The analysis of such materials requires incorporating the

theories of oriented media. For this reason, micropolar theories were developed by Eringen(1966a,b) for elastic solids and fluids and are now universally accepted.

Following various methods, the elastic fields of various loadings, inclusion and inhomogeneity problems, and interaction energy of point defects and dislocation arrangement have been discussed extensively in the past. Generally all materials have elastic anisotropic properties which mean the mechanical behavior of an engineering material is characterized by the direction dependence. However the three dimensional study for an anisotropic material is much more complicated to obtain than the isotropic one, due to the large number of elastic constants involved in the calculation.

Because a wide class of crystals such as W, Si, Cu, Ni, Fe, Au, Al etc., which are some frequent used substances, belong to cubic materials. The cubic materials have nine planes of symmetry whose normals are on the three coordinate axes and on the coordinate planes making an angle $\pi/4$ with the coordinate axes. With the chosen coordinate system along the crystalline directions, the mechanical behavior of a micropolar cubic crystal can be characterized by four independent elastic constants.

To understand the crystal elasticity of a cubic material, Chung and Buessem (1967) presented a convenient method to describe the degree of the elasticity anisotropy in a given cubic crystal. Later, Lie and Koehler (1968) used a Fourier expansion scheme to calculate the stress fields caused by a unit force in a cubic crystal. Steeds (1973) gave a complete discussion on the displacements, stresses and energy factors of the dislocations for two-dimensional anisotropic materials. Boulanger and Hayes (2000) investigated inhomogeneous plane waves in cubic elastic materials. Bertram et al. (2000) discussed generation of discrete isotropic orientation distributions for linear elastic cubic crystals. Kobayashi et al. (2001) investigated anisotropy and curvature effects for growing crystals. Domanski and Jablonski (2001) studied resonances of nonlinear elastic waves in cubic crystal. Destrade (2001) considered the explicit secular equation for surface acoustic waves in monoclinic elastic crystals. Zhou and Ogawa (2002) investigated elastic solutions for a solid rotating disk with cubic anisotropy. Minagawa et al. (1981) discussed the propagation of plane harmonic waves in a cubic micropolar medium. Kumar and Rani (2003) studied time harmonic sources in a thermally conducting cubic crystal. Recently, Kumar and Ailawalia (2005a, 2005b) discussed source problems in micropolar cubic crystals.

Kuo (1969) and Garg *et.al* (2003) discussed the problem of inclined load in the theory of elastic solids. The deformation due to other sources such as strip loads, continuous line loads, etc. can also be similarly obtained. The deformation at any point of the medium is useful to analyze the deformation field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region. No attempt has been made so far to study the response of inclined load in micropolar theory of elasticity.

2. Problem Formulation

We consider a homogeneous micropolar cubic crystal of infinite extent with Cartesian coordinate system (x, y, z). To analyze displacement and stresses at the interior of the medium due to inclined load, the continuum is divided into two half-spaces defined by:

- (i) half space I $|x| < \infty$, $-\infty < y \le 0$, $|z| \le \infty$,
- (ii) half space II $|x| < \infty$, $0 \le y < \infty$, $|z| \le \infty$.

Suppose that an inclined line load F_0 is acting on the z-axis and its inclination with y-axis is θ .



Figure 1 Continuum under load F_0

If we restrict our analysis to plane strain parallel to x-y plane with displacement vector $\vec{u} = (u_1, u_2, 0)$ and microrotation vector $\vec{\phi} = (0, 0, \phi_3)$ then the field equations and constitutive relations for such a medium in the absence of body forces and body couples can be written by following the equations given by Minagawa *et al.*(1981) as

$$A_1 \frac{\partial^2 u_1}{\partial x^2} + A_3 \frac{\partial^2 u_1}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 u_2}{\partial x \partial y} + (A_3 - A_4) \frac{\partial \phi_3}{\partial y} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad (1)$$

$$A_3 \frac{\partial^2 u_2}{\partial x^2} + A_1 \frac{\partial^2 u_2}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 u_1}{\partial x \partial y} - (A_3 - A_4) \frac{\partial \phi_3}{\partial x} = \rho \frac{\partial^2 u_2}{\partial t^2}, \qquad (2)$$

$$B_3 \nabla^2 \phi_3 + (A_3 - A_4) \left(\frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \right) - 2 (A_3 - A_4) \phi_3 = \rho j \frac{\partial^2 \phi_3}{\partial t^2}, \qquad (3)$$

$$t_{22} = A_2 \frac{\partial u_1}{\partial x} + A_1 \frac{\partial u_2}{\partial y}, \qquad (4)$$

$$t_{21} = A_4 \left(\frac{\partial u_2}{\partial x} - \phi_3\right) + A_3 \left(\frac{\partial u_1}{\partial y} + \phi_3\right), \tag{5}$$

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$$m_{23} = B_3 \frac{\partial \phi_3}{\partial y} \,. \tag{6}$$

In these relations, we have used the following notations: t_{22}, t_{21} – components of the force stress tensor, m_{23} – tangential couple stress, u_i – components of displacement vector, ϕ_3 – component of microrotation vector, A_1, A_2, A_3, A_4, B_3 – characteristic constants of the material, ρ – the density and j – the microinertia.

Introducing the dimensionless variables defined by the expressions

$$x' = \frac{\omega^*}{c_1}x, \quad y' = \frac{\omega^*}{c_1}y, \quad u'_1 = \frac{\omega^*}{c_1}u_1, \quad u'_2 = \frac{\omega^*}{c_1}u_2, \quad \phi'_3 = \frac{A_1}{A_4}\phi_3, \quad t' = \omega^*t,$$

$$\{t'_{22}, t'_{21}\} = \frac{\{t_{22}, t_{21}\}}{A_1}, \quad m'_{23} = \frac{c_1}{B_3\omega^*}m_{23}, \quad \{F'_1, F'_2\} = \frac{\{F_1, F_2\}}{A_1} \quad a' = \frac{\omega^*}{c_1}a.$$
(7)

where

$$\omega^{*2} = \frac{A_4 - A_3}{\rho j} \quad , \quad c_1^2 = \frac{A_1}{\rho} \tag{8}$$

Using (7), the system of equations (1)-(3) reduce to(dropping the primes)

$$A_1 \frac{\partial^2 u_1}{\partial x^2} + A_3 \frac{\partial^2 u_1}{\partial y^2} + (A_2 + A_4) \frac{\partial^2 u_2}{\partial x \partial y} + \frac{A_4 (A_3 - A_4)}{A_1} \frac{\partial \phi_3}{\partial y} = \rho c_1^2 \frac{\partial^2 u_1}{\partial t^2}, \quad (9)$$

$$A_{3}\frac{\partial^{2}u_{2}}{\partial x^{2}} + A_{1}\frac{\partial^{2}u_{2}}{\partial y^{2}} + (A_{2} + A_{4})\frac{\partial^{2}u_{1}}{\partial x\partial y} - \frac{A_{4}(A_{3} - A_{4})}{A_{1}}\frac{\partial\phi_{3}}{\partial x} = \rho c_{1}^{2}\frac{\partial^{2}u_{2}}{\partial t^{2}}, \quad (10)$$

$$B_{3}\frac{A_{4}\omega^{*2}}{A_{1}c_{1}^{2}}\nabla^{2}\phi_{3} + (A_{3} - A_{4})\left(\frac{\partial u_{2}}{\partial x} - \frac{\partial u_{1}}{\partial y}\right) - 2\frac{A_{4}(A_{3} - A_{4})}{A_{1}}\phi_{3} = \rho j\omega^{*2}\frac{A_{4}}{A_{1}}\frac{\partial^{2}\phi_{3}}{\partial t^{2}}.$$
(11)

We suppose that initially the medium is at rest in its undeformed state, i.e, we suppose that the following homogeneous initial conditions hold for $t \ge 0$:

$$u_i(x, y, 0) = \frac{\partial u_i}{\partial t} = 0, \quad \phi_3(x, y, 0) = \frac{\partial \phi_3}{\partial t} = 0,$$

Applying Laplace transform with respect to time 't' defined by

$$\left\{\overline{u}_{n}(x,y,p), \overline{\phi}_{3}(x,y,p)\right\} = \int_{0}^{\infty} \left\{u_{n}(x,y,t), \phi_{3}(x,y,t)\right\} e^{-pt} dt, n = 1, 2$$
(12)

and then Fourier transform with respect to 'x' defined by

$$\left\{\tilde{u}_{n}\left(\xi,y,p\right),\tilde{\phi}_{3}\left(\xi,y,p\right)\right\}=\int_{-\infty}^{\infty}\left\{\overline{u}_{n}\left(x,y,p\right),\overline{\phi}_{3}\left(x,y,p\right)\right\}\,e^{i\xi x}\,dx\,,\,n=1,2\tag{13}$$

on the equations (9)-(11) with the help of initial conditions, we obtain

$$D^{2}\tilde{u}_{1} = b_{11}\,\tilde{u}_{1} + a_{12}D\,\tilde{u}_{2} + a_{13}\,D\tilde{\phi}'_{3}\,,\tag{14}$$

$$D^2 \tilde{u}_2 = b_{22} \,\tilde{u}_2 + a_{21} \, D \tilde{u}_1 + b_{23} \,\tilde{\phi}_3 \,, \tag{15}$$

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$$D^2 \tilde{\phi}_3 = b_{33} \, \tilde{\phi}_3 + a_{31} D \, \tilde{u}_1 + b_{32} \, \tilde{u}_2 \quad , \tag{16}$$

and

$$b_{11} = \frac{\rho c_1^2 p^2 + \xi^2 A_1}{A_3}, \quad b_{22} = \frac{\rho c_1^2 p^2 + \xi^2 A_3}{A_1}, \quad b_{23} = -\frac{i\xi A_4 (A_3 - A_4)}{A_1^2},$$

$$b_{32} = \frac{i\xi c_1^2 A_1 (A_3 - A_4)}{\omega^{*2} A_4 B_3}, \quad b_{33} = \frac{1}{B_3} \left[\xi^2 B_3 + \rho j c_1^2 p^2 + 2 (A_3 - A_4) \frac{c_1^2}{\omega^{*2}} \right], \quad (17)$$

$$a_{13} = -\frac{A_4 (A_3 - A_4)}{A_1 A_3}, \quad a_{21} = \frac{i\xi (A_2 + A_4)}{A_1},$$

$$a_{31} = \frac{A_1 (A_3 - A_4) c_1^2}{A_4 B_3 \omega^{*2}}, \quad a_{12} = \frac{i\xi (A_2 + A_4)}{A_3}, \quad D = \frac{d}{dy}.$$

The equations (14)–(16) may be written as

$$\frac{d}{dy} W(\xi, y, p) = A(\xi, p) \quad W(\xi, y, p) .$$
(18)

where

$$W = \begin{bmatrix} V \\ DV \end{bmatrix}, \quad A = \begin{bmatrix} O & I \\ A_1^* & A_2^* \end{bmatrix}, \quad V = \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \\ \tilde{\phi}_3 \end{bmatrix},$$
$$A_1^* = \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & b_{23} \\ 0 & b_{32} & b_{33} \end{bmatrix}, \quad A_2^* = \begin{bmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{bmatrix}.$$
(19)

 ${\bf O}$ and ${\bf I}$ are respectively zero and identity matrix of order 3.

To solve equation (18), we assume

$$W(\xi, y, p) = X(\xi, p) \quad e^{qy} \tag{20}$$

which leads to eigenvalue problem. The characteristic equation corresponding to matrix \mathbf{A} is given by

$$\det \left[\mathbf{A} - q\mathbf{I} \right] = 0 \tag{21}$$

which on expansion provides us

$$q^{6} + \lambda_{1}q^{4} + \lambda_{2}q^{2} + \lambda_{3} = 0 \tag{22}$$

where

$$\lambda_1 = -(a_{12}a_{21} + a_{13}a_{31} + b_{11} + b_{22} + b_{33})$$

 $\lambda_2 = a_{12} \left(a_{21} b_{33} - b_{23} a_{31} \right) + a_{13} \left(b_{22} a_{31} - a_{21} b_{32} \right) + b_{22} b_{33} - b_{23} b_{32} + b_{11} \left(b_{22} + b_{33} \right) \,,$

$$\lambda_3 = b_{11} \left(b_{23} b_{32} - b_{22} b_{33} \right) \,. \tag{23}$$

The eigenvalues of the matrix **A** are characteristic roots of the equation (22). The vectors $X(\xi, p)$ corresponding to the eigenvalues q_s can be determined by solving the homogeneous equation

$$[\mathbf{A} - q\mathbf{I}] \ X(\xi, p) = 0 \tag{24}$$

The set of eigenvectors $X_{s}\left(\xi,p\right)$, $s=1,2,\ldots,6$ may be obtained as

$$X_s(\xi, p) = \begin{bmatrix} X_{g1}(\xi, p) \\ X_{g2}(\xi, p) \end{bmatrix}$$
(25)

where

$$X_{g1}(\xi, p) = \begin{bmatrix} q_g \\ a_g \\ b_g \end{bmatrix},$$

$$X_{g2}(\xi, p) = \begin{bmatrix} q_g^2 \\ a_g q_g \\ b_g q_g \end{bmatrix}, \quad q = q_g ; g = 1, 2, 3$$
(26)

$$\begin{aligned} X_{R1}(\xi, p) &= \begin{bmatrix} -q_R \\ a_R \\ b_R \end{bmatrix}, \\ X_{R2}(\xi, p) &= \begin{bmatrix} q_R^2 \\ -a_R q_R \\ -b_R q_R \end{bmatrix}, \quad R = g + 3 \ ; \ q = -q_g \ ; g = 1, 2, 3 \end{aligned}$$

and

$$a_g = \frac{b_{11}b_{23} - q_g^2 (b_{23} + a_{21}a_{13})}{\nabla_g},$$

$$b_g = \frac{q_g^2 a_{31} + a_g b_{32}}{q_g^2 - b_{33}},$$

$$\nabla_g = q_g^2 a_{13} + a_{12}b_{23} - b_{22}a_{13}.$$
(27)

The solution of equation (20) is given by

$$W(\xi, y, p) = \sum_{s=1}^{3} \left[D_s X_s(\xi, p) \exp(q_s y) + D_{s+3} X_{s+3}(\xi, p) \exp(-q_s y) \right].$$
(28)

3. Application

We consider a normal line load F_1 acting in the positive y direction on the interface y = 0 along the z-axis and a line force F_2 acting at the origin in the positive x direction, then the boundary conditions at the interface of two half spaces (y = 0) are given by,

$$u_{1}(x,0^{+},t) - u_{1}(x,0^{-},t) = 0$$

$$u_{2}(x,0^{+},t) - u_{2}(x,0^{-},t) = 0$$

$$\phi_{3}(x,0^{+},t) - \phi_{3}(x,0^{-},t) = 0$$

$$t_{22}(x,0^{+},t) - t_{22}(x,0^{-},t) = -F_{1}\psi(x)\delta(t)$$

$$t_{21}(x,0^{+},t) - t_{21}(x,0^{-},t) = -F_{2}\psi(x)\delta(t)$$

$$m_{23}(x,0^{+},t) - m_{23}(x,0^{-},t) = 0$$

(29)



Figure 2 Continuum under loads F_1 and F_2

Using equations (7) and then applying Laplace and Fourier transforms from equations (12) and (13) on system of equations (29) and with the help of equations (28), we get the transformed displacement and stresses as,

$$\begin{split} \tilde{u}_{1} &= -\frac{1}{\Delta} \begin{bmatrix} q_{1}\Delta_{1}e^{-q_{1}y} + q_{2}\Delta_{2}e^{-q_{2}y} + q_{3}\Delta_{3}e^{-q_{3}y} - q_{1}\Delta_{4}e^{q_{1}y} - q_{2}\Delta_{5}e^{q_{2}y} - q_{3}\Delta_{6}e^{q_{3}y} \end{bmatrix}, \\ \tilde{u}_{2} &= \frac{1}{\Delta} \begin{bmatrix} a_{1}\Delta_{1}e^{-q_{1}y} + a_{2}\Delta_{2}e^{-q_{2}y} + a_{3}\Delta_{3}e^{-q_{3}y} + a_{1}\Delta_{4}e^{q_{1}y} + a_{2}\Delta_{5}e^{q_{2}y} + a_{3}\Delta_{6}e^{q_{3}y} \end{bmatrix}, \\ \tilde{t}_{21} &= \frac{1}{\Delta} \begin{bmatrix} s_{1}\Delta_{1}e^{-q_{1}y} + s_{2}\Delta_{2}e^{-q_{2}y} + s_{3}\Delta_{3}e^{-q_{3}y} - s_{1}\Delta_{4}e^{q_{1}y} - s_{2}\Delta_{5}e^{q_{2}y} - s_{3}\Delta_{6}e^{q_{3}y} \end{bmatrix}, \\ \tilde{t}_{22} &= \frac{1}{\Delta} \begin{bmatrix} r_{1}\Delta_{1}e^{-q_{1}y} + r_{2}\Delta_{2}e^{-q_{2}y} + r_{3}\Delta_{3}e^{-q_{3}y} + r_{1}\Delta_{4}e^{q_{1}y} + r_{2}\Delta_{5}e^{q_{2}y} + r_{3}\Delta_{6}e^{q_{3}y} \end{bmatrix}, \\ (33) \end{split}$$

$$\tilde{m}_{23} = -\frac{A_4}{A_1\Delta} \left[b_1 q_1 \Delta_1 e^{-q_1 y} + b_2 q_2 \Delta_2 e^{-q_2 y} + b_3 q_3 \Delta_3 e^{-q_3 y} - b_1 q_1 \Delta_4 e^{q_1 y} - b_2 q_2 \Delta_5 e^{q_2 y} - b_3 q_3 \Delta_6 e^{q_3 y} \right],$$
(34)

where

$$\begin{split} \Delta &= 8GH \;, \quad G = s_1 \left(a_2 b_3 - a_3 b_2 \right) - s_2 \left(a_1 b_3 - a_3 b_1 \right) + s_3 \left(a_1 b_2 - a_2 b_1 \right) \;, \\ H &= r_1 q_2 q_3 \left(b_3 - b_2 \right) - r_2 q_1 q_3 \left(b_3 - b_1 \right) + r_3 q_1 q_2 \left(b_2 - b_1 \right) \;, \\ \Delta_{1,4} &= -4G q_2 q_3 \left(b_3 - b_2 \right) F_1 \tilde{\psi} \left(x \right) \pm 4H \left(a_2 b_3 - a_3 b_2 \right) F_2 \tilde{\psi} \left(\xi \right) \;, \\ \Delta_{2,5} &= 4G q_1 q_3 \left(b_3 - b_1 \right) F_1 \tilde{\psi} \left(x \right) \mp 4H \left(a_1 b_3 - a_3 b_1 \right) F_2 \tilde{\psi} \left(\xi \right) \;, \\ \Delta_{3,6} &= -4G q_1 q_2 \left(b_2 - b_1 \right) F_1 \tilde{\psi} \left(x \right) \pm 4H \left(a_1 b_2 - a_2 b_1 \right) F_2 \tilde{\psi} \left(\xi \right) \;, \\ r_\Theta &= q_\Theta \left(i\xi \frac{A_2}{A_1} - a_\Theta \right) \;, \\ s_\Theta &= \frac{1}{A_1} \left[-i\xi A_4 a_\Theta + A_3 q_\Theta^2 + \left(A_3 - A_4 \right) \frac{A_4}{A_1} b_\Theta \right] \;; \; \Theta = 1, 2, 3 \;. \end{split}$$
(35)

3.1. Concentrated load

In order to determine displacements, microrotation and stresses due to concentrated load described as Dirac delta function $\psi(x) = \delta(x)$ must be used when concentrated force is applied. The Fourier transform of $\psi(x)$ with respect to pair (x, ξ) will be $\tilde{\psi}(\xi) = 1$.

3.2. Uniformly distributed load

The solution due to distributed load applied is obtained by setting

$$\psi\left(x\right) = \begin{bmatrix} 1 & if & |x| \le a, \\ 0 & if & |x| > a, \end{bmatrix}$$

in equations (30). The Fourier transform with respect to the pair (x, ξ) for the case of a uniform strip load of unit amplitude and width 2a applied at the origin of the coordinate system (x = y = 0) in dimensionless form after suppressing the primes becomes

$$\tilde{\psi}\left(\xi\right) = \left[2\,Sin\left(\frac{\xi c_1 a}{\omega^*}\right)/\xi\right] \,, \quad \xi \neq 0 \,. \tag{36}$$

3.3. Linearly distributed load

The solution due to linearly distributed load is obtained by substituting

$$\psi(x) = \begin{bmatrix} 1 - \frac{|x|}{a} & if \quad |x| \le a, \\ 0 & if \quad |x| > a, \end{bmatrix}$$
(37)

The Fourier transform in case of linearly distributed loads applied at the origin of the system in dimensionless form are

$$\tilde{\psi}\left(\xi\right) = \frac{2\left[1 - \cos\left(\frac{\xi c_1 a}{\omega^*}\right)\right]}{\frac{\xi^2 c_1 a}{\omega^*}} \,. \tag{38}$$

3.4. Inclined line load

For an inclined line load F_0 we have (see Figure 1)

$$F_1 = F_0 \cos \theta , \quad F_2 = F_0 \sin \theta . \tag{39}$$

Using (39) and (3.1)–(3.3) in (30)–(34), we obtain the corresponding expressions for displacement and stress components in case of concentrated, uniformly distributed and linearly distributed inclined load applied at any point in micropolar cubic crystal.

4. Particular case

Taking

$$A_1 = \lambda + 2\mu + K$$
, $A_2 = \lambda$, $A_3 = \mu + K$, $A_4 = \mu$, $B_3 = \gamma$,

we obtain the corresponding expressions for the micropolar isotropic medium. These results tally with the one if we solve the problem in micropolar isotropic medium.

5. Inversion of the transform

The transformed displacements and stresses are functions of y, the parameters of Laplace and Fourier transforms p and ξ respectively, and hence are of the form $\tilde{f}(\xi, y, p)$. To get the function in the physical domain, first we invert the Fourier transform using

$$\overline{f}(x, y, p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-i\xi x} \widetilde{f}(\xi, y, p) d\xi,$$
$$= \frac{1}{\pi} \int_{0}^{\infty} \{\cos\left(\xi x\right) f_e - i\sin\left(\xi x\right) f_o\} d\xi, \qquad (40)$$

where f_e and f_o are even and odd parts of the function $\tilde{f}(\xi, y, p)$ respectively. Thus, expressions (31)–(33) give us the transform $\overline{f}(x, y, p)$ of the function f(x, y, t).

Following Honig and Hirdes (1984) the Laplace transform function $\overline{f}(x, y, p)$ can be inverted to f(x, y, t).

The last step is to evaluate the integral in equation (38). The method for evaluating this integral by Press *et al.* (1986) and which involves the use of Rhomberg's integration with adaptive step size. This, also uses the results from successive refinement of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

6. Numerical results and discussions

For numerical computations, we take the following values of relevant parameters for micropolar cubic crystal as,

$$\begin{split} A_1 &= 13.97 \times 10^{10} \, \rm{dyne/cm}^2 \\ A_3 &= 3.2 \times 10^{10} \, \rm{dyne/cm}^2 \ , A_2 &= 13.75 \times 10^{10} \, \rm{dyne/cm}^2 \ , \\ A_4 &= 2.2 \ \times \ 10^{10} \, \rm{dyne/cm}^2 \ , B_3 &= 0.056 \ \times 10^{10} \rm{dynes} \ . \end{split}$$

For the comparison with micropolar isotropic solid, following Gauthier(1982), we take the following values of relevant parameters for the case of Aluminium epoxy composite as:

$$\begin{split} \rho &= 2.19\,\mathrm{gm/cm^3}\ , \quad \lambda = 7.59\times 10^{10}\,\mathrm{dyne/cm^2}\ , \quad \mu = 1.89\times 10^{10}\,\mathrm{dyne/cm^2}\ , \\ K &= 0.0149\times 10^{10}\,\mathrm{dyne/cm^2}\ , \quad \gamma = 0.0268\times 10^{10}\,\mathrm{dyne}\ , \quad j = 0.00196\,\,\mathrm{cm^2}\ . \end{split}$$

The values of tangential displacement $U_1 = (u_1/F_0)$, normal displacement $U_2 = (u_2/F_0)$, tangential force stress $T_{21} = (t_{21}/F_0)$ normal force stress $T_{22} = (t_{22}/F_0)$ and tangential couple stress $M_{23} = (m_{23}/F_0)$ for a micropolar cubic crystal (MCC) and micropolar isotropic solid (MIS) have been studied and the variations of these components with distance x have been shown by

- solid line (-----) for MCC and $\theta = 0^0$,
- dashed line (----) for MIS and $\theta = 0^0$,

- solid line with centered symbol (x-x-x-x) for MCC and $\theta = 30^0$,
- dashed line with centered symbol(x-x-x) for MIS and $\theta = 30^0$,
- solid line with centered symbol (o⁻⁻o⁻⁻o⁻⁻o) for MCC and $\theta = 60^0$,
- dashed line with centered symbol(o—o—o) for MIS and $\theta = 60^{\circ}$,
- solid line with centered symbol (* * * *) for MCC and $\theta = 90^{\circ}$,
- dashed line with centered symbol($_*-_*-_*$) for MIS and $\theta = 90^0$.

These variations are shown in Figures 3–17. The computations are carried out for y = 1.0 in the range $0 \le x \le 10.0$ and a = 1.0 at t = 0.1.

7. Discussions

7.1. Concentrated Load

The variations of normal components of displacement and force stress are more oscillatory as compared to their tangential counterparts. Also for a particular inclination of load, the values of displacement components (both normal and tangential), near the point of application of source, are more for MIS as compared to the values for MCC. But the same is not true in case of force stress where the values are more for MCC in comparison to the values for MIS, close to the point of application of source. Also it is interesting to observe that the magnitude of oscillations of all the quantities decreases with increase in angle of inclination of the source with normal direction. The variations of tangential displacement, normal displacement, tangential force stress and normal force stress for MCC and MIS and for different inclinations are shown in Figures 3–6 respectively.

The variations of tangential couple stress for both MCC and MIS rises initially and then oscillates with increase in distance x. However the magnitude of these oscillations decreases with increase in angle of inclination of the source with normal direction. These variations of tangential couple stress are shown in Figure 7.

7.2. Uniformly distributed load

It is observed that the normal components of displacement and force stress are more oscillatory as compared to tangential components. Moreover the values of all the quantities are more for MIS and hence to compare the variations among both solids, the values of all the quantities for MIS have been demagnified by 100. The values of tangential and normal components of displacement and force stress, very close to the point of application of source, increases with increase in angle of inclination of the applied source to the normal direction. These variations of tangential displacement, normal displacement, tangential force stress and normal force stress are shown in Figures 8-11 respectively.

The values of tangential couple stress for MCC lie in a short range as compared to the values for MIS. Also the variations of tangential couple stress for MIS traces a bell shaped region and the area under the region decreases with increase in orientation of applied source. The values of tangential couple stress for MIS have been demagnified by 100 and these variations are shown in Figure 12.



Figure 3 Variation of tangential displacement $U_1=\frac{u_1}{F_0}$ with distance x for concentrated load at t=0.1



Figure 4 Variation of normal displacement $U_2=\frac{u_2}{F_0}$ with distance x for concentrated load at t=0.1



Figure 5 Variation of tangential force stress $T_{21} = \frac{t_{21}}{F_0}$ with distance x for concentrated load at t = 0.1



Figure 6 Variation of normal force stress $T_{22}=\frac{t_{22}}{F_0}$ with distance x for concentrated load at t=0.1



Figure 7 Variation of tangential couple stress $M_{23} = \frac{m_{23}}{F_0}$ with distance x for concentrated load at t = 0.1



Figure 8 Variation of tangential displacement $U_1=\frac{u_1}{F_0}$ with distance x for uniformly distributed load at t=0.1



Figure 9 Variation of normal displacement $U_2 = \frac{u_2}{F_0}$ with distance x for concentrated load at t = 0.1



Figure 10 Variation of tangential force stress $T_{21} = \frac{t_{21}}{F_0}$ with distance x for uniformly distributed load at t = 0.1



Figure 11 Variation of normal force stress $T_{22} = \frac{t_{22}}{F_0}$ with distance x for uniformly distributed load at t = 0.1



Figure 12 Variation of tangential couple stress $M_{23}=\frac{m_{23}}{F_0}$ with distance x for uniformly distributed load at t=0.1



Figure 13 Variation of tangential displacement $U_1=\frac{u_1}{F_0}$ with distance x for linearly distributed load at t=0.1



Figure 14 Variation of normal displacement $U_3 = \frac{u_3}{F_0}$ with distance x for linearly distributed load at t = 0.1



Figure 15 Variation of tangential force stress $T_{31} = \frac{t_{31}}{F_0}$ with distance x for linearly distributed load at t = 0.1



Figure 16 Variation of normal force stress $T_{22} = \frac{t_{22}}{F_0}$ with distance x for linearly distributed load at t = 0.1



Figure 17 Variation of tangential couple stress $M_{23} = \frac{m_{23}}{F_0}$ with distance x for linearly distributed load at t = 0.1

7.3. Linearly distributed load

The variations of tangential displacement for both MCC and MIS are opposite in nature. While the variation increases with horizontal distance for MCC, it decreases for MIS. These variations are shown in Figure 13. The variations of normal displacement are quite similar to the variations of tangential displacement with the exception that while the values of normal displacement decreases with increase in distance x, it remains constant (to some extent) in the range $0 \le x \le 4.0$ for MIS(for different orientations). These variations of normal displacement are shown in Figure 14.

It is observed that the values of tangential and normal force stress increases with increase in distance x for both MCC and MIS but this rise is more sharp and more uniform for tangential force stress. Moreover the intensity of sharpness decreases as the source moves from normal to tangential direction. These variations of tangential and normal force stress are shown in Figures 15 and 16 respectively.

The values of tangential couple stress for both MCC and MIS decreases sharply with horizontal distance. Also these values are less for MCC as compared to the values for MIS. The values of tangential couple stress for MIS have been demagnified by 10 in order to compare the results among both the solids. The variations of tangential couple stress have been shown in Figure 17.

8. Conclusion

The properties of a body depend largely on the direction of symmetry. The normal components of displacement and force stress are more oscillatory as compared to their tangential counterparts. Also the magnitude of oscillations for both MCC and MIS decreases as the applied source moves from normal to tangential direction. It is observed that the body is deformed to a more extent on the application of strip loading particularly when linearly distributed load is applied.

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