Mechanics and Mechanical Engineering Vol. 14, No. 1 (2010) 65–79 © Technical University of Lodz

# Dynamics Modeling and Identification of the Amigobot Robot

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> Received (10 January 2010) Revised (27 February 2010) Accepted (10 March 2010)

This paper presents problems connected with the identification of dynamic motion equations derived on the basis of Maggi equations for a 2–wheeled mobile robot. The possibility of sending data between  $Maple^{TM}$  and  $Matlab^{TM}$  has been discussed. Off–line identification structure has been presented with the use of genetic algorithms.

Keywords: Dynamics, identification, fuzzy logic, mobile robots

### 1. Introduction

The basic problem of the analysis of the complex dynamic constructions is the mathematical model description and its identification. "Identifying" means the choice of the best model in the sense of the accepted quality criterion of the class of models formed on the basis of mathematical description of physical phenomena [1]. In this paper the problem connected with mathematical model description and parameters identification of the dynamic motion equations derived on the basis of Maggi equations for mobile robot AmigoBot<sup>TM</sup> has been presented. The paper suggest the mode of symbolic computations in the environment of Maple<sup>TM</sup> programme and afterwards the moving of the data to the Matlab<sup>TM</sup> package. In the Matlab<sup>TM</sup>– Simulink environment simulation of the robot's dynamic behaviour has been carried out. Such a mode of computations and communicative Matlab<sup>TM</sup>–Maple<sup>TM</sup> software have been further discussed in paper [2]. On the basis of kinematic and dynamic parameters measured on real robot test rigs connected with identification have been carried out.

# 2. Modeling of the dynamics of the wheeled mobile robot AmigoBot $_{TM}$

The analysis of the dynamics has been carried out for AmigoBot<sup>TM</sup> robot. The model of this robot has been shown in a schematic mode in Fig. 1. The basic elements of the model are: wheel unit drive of wheels 1 and 2, self-adjusting supporting wheel 3 and the frame of unit 4. As a result of symbolic computations dynamic equations for the model taken have been received. In these equations the influence of the mass of the self-adjusting supporting wheel has not been taken into consideration [1,2,3].



Figure 1 Computation model of  $\operatorname{AmigoBot}^{TM}$  robot

There are many mathematical forms of the dynamic motion equations descriptions. One of the method based on Lagrange description is has been used to describe motion of the mobile robot. This mathematical form of the dynamic motion equations is called as Maggi equations. This method, in many solutions, is more efficient and easier to describe dynamics of the system. Additionally when we are describing system receiving equations in quantity of the DOF of this mechanical structure. The mathematical form of Maggi equations has been presented below [4,5,7]:

$$\sum_{j=1}^{n} C_{ij} \left[ \frac{d}{dt} \left( \frac{\partial E}{\partial \dot{q}_j} \right) - \left( \frac{\partial E}{\partial q_j} \right) \right] = \Theta_i \qquad i = 1..s$$
(1)

Where s describe quantity of the independent system parameters in generalized coordinates  $q_j$  (j=1..n) in the quantity equal DOF of the system. On the basis of Maggi description we can present generalized velocity formula in the form:

$$\dot{q}_j = \sum_{i=1}^{s} C_{ij} \dot{e}_i + G_j$$
 (2)

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In the equation (2) parameter  $\dot{e}_i$  is called characteristic or kinematics of the system in generalized coordinates. The right sides of the Maggi equations (1) this are coefficients connected with variance  $\delta e_i$  in description of the virtual work of the external forces of the system. Definition of the virtual work of the external forces can be presented as follow [1,2]

$$\sum_{i=1}^{s} \Theta_i \delta e_i = \sum_{i=1}^{s} \delta e_i \sum_{i=1}^{n} C_{ij} Q_j \tag{3}$$

In the form of the matrix presentation, equations (1) have been presented as follow:

$$\sum_{j=1}^{n} C_{ij} L_j = \Theta_i \qquad i = 1..s \tag{4}$$

Where [7]:

$$L = M(q)\ddot{q} + C(q,\dot{q})\dot{q} \tag{5}$$

Maggiego equations (1) as Lagrange equations can be used to analyze the task simple and inverse dynamics. Taking into account the problem of control, it appears that the use of Maggi equations seems to be easer to described. The mathematical forms of the Maggi and Lagrange equations can be also apply in the case of all other stationary or wheeled mobile robots.

To determine the dynamic equations of motion adopted following vector of generalized coordinates:

$$q = [x_A, y_A, \beta, \alpha_1, \alpha_2]^T \tag{6}$$

The kinetics energy of the model illustrated in Fig. 1. as a function of generalized coordinates without taking into account the influence of the self-adjusting supported wheel 3 has been recorded as follows:

$$E_{k} = [(m_{1} + m_{2} + m_{4})\dot{x}_{A} + ((m_{1} - m_{2})l_{1}\cos(\beta) + m_{4}l_{2}\sin(\beta))\beta]\dot{x}_{A} + [(m_{1} + m_{2} + m_{4})\dot{y}_{A} + ((m_{1} - m_{2})l_{1}\sin(\beta) - m_{4}l_{2}\cos(\beta))\dot{\beta}]\dot{y}_{A} + [((m_{1} - m_{2})l_{1}\cos(\beta) + m_{4}l_{2}\sin(\beta))\dot{x}_{A} + ((m_{1} - m_{2})l_{1}\sin(\beta) (7) - m_{4}l_{2}\cos(\beta))\dot{y}_{A} + ((m_{1} + m_{2})l_{1}^{2} + Ix_{1} + Ix_{2} + Iz_{4} + m_{4}l_{2}^{2})\dot{\beta}]\dot{\beta} + [Iz_{1}\dot{\alpha}_{1}]\dot{\alpha}_{1} + [Iz_{2}\dot{\alpha}_{2}]\dot{\alpha}_{2}$$
(7)

where:

 $m_1, m_2, m_3, m_4$  – weight of each component in the system

 $Ix_1, Ix_2, Ix_3, Iz_1, Iz_2, Iz_3, Iz_4$  – moments of inertia with respect to the axes h – parameter defining the ratio  $l_1/r_1$ 

In the same way as in the Lagrange equations, matrices of the inertia and centrifugal forces of inertia and the Coriolis for the model the calculation have been determined. Form of the matrix is shown below:

$$M = \begin{bmatrix} 2m_1 + m_4 & 0 & m_4 l_2 \sin(\beta) & 0 & 0\\ 0 & 2m_1 + m_4 & -m_4 l_2 \cos(\beta) & 0 & 0\\ m_4 l_2 \sin(\beta) & -m_4 l_2 \cos(\beta) & 2m_1 l_1^2 + 2Ix_1 + Iz_4 + m_4 l_2^2 & 0 & 0\\ 0 & 0 & 0 & Iz_1 \end{bmatrix}$$

$$(8)$$

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With the appointment of external forces acting on the system should take into account the unknown dry friction forces acting in the plane of wheel contact with the road and acting on appropriate wheels [1,2]. Fig. 2 present the distribution of the friction forces acting on wheels of a mobile wheeled robot.



Figure 2 The distribution of friction forces acting on the wheeled mobile robot

Generalized forces for the model shown in rys.3.2. described as follow:

$$Q_{1} = T_{1,0} \cos(\beta) - T_{1,P} \sin(\beta) + T_{2,0} \cos(\beta) - T_{2,P} \sin(\beta)$$

$$Q_{2} = T_{1,0} \sin(\beta) + T_{1,P} \cos(\beta) + T_{2,0} \sin(\beta) + T_{2,P} \cos(\beta)$$

$$Q_{3} = T_{1,0}l_{1} - T_{2,0}l_{1}$$

$$Q_{4} = M_{1} - N_{1}f_{1}sgn(\dot{\alpha}_{1}) - T_{1,0}r$$

$$Q_{5} = M_{2} - N_{2}f_{2}sgn(\dot{\alpha}_{2}) - T_{2,0}r$$
(10)

The generalized velocities based on dependence (2) states:

$$\dot{q}_{1} = \dot{x}_{A} = \frac{1}{2}r\dot{e}_{1}\cos(\beta) + \frac{1}{2}r\dot{e}_{2}\cos(\beta)$$

$$\dot{q}_{2} = \dot{y}_{A} = \frac{1}{2}r\dot{e}_{1}\sin(\beta) + \frac{1}{2}r\dot{e}_{2}\sin(\beta)$$

$$\dot{q}_{3} = \dot{\beta} = \frac{1}{2}\frac{r}{l_{1}}\dot{e}_{1} - \frac{1}{2}\frac{r}{l_{1}}\dot{e}_{2}$$

$$\dot{q}_{4} = \dot{\alpha}_{1} = \dot{e}_{1}$$

$$\dot{q}_{5} = \dot{\alpha}_{2} = \dot{e}_{2}$$
(11)

The  $C_{ij}$  and  $G_j$  coefficients from the equation (2) have the form:

$$C_{11} = \frac{1}{2}r\cos(\beta), \quad C_{21} = \frac{1}{2}r\cos(\beta), \quad G_1 = 0$$

$$C_{12} = \frac{1}{2}r\sin(\beta), \quad C_{22} = \frac{1}{2}r\sin(\beta), \quad G_2 = 0$$

$$C_{13} = \frac{1}{2}\frac{r}{l_1}, \quad C_{22} = -\frac{1}{2}\frac{r}{l_1}, \quad G_3 = 0$$

$$C_{14} = 1, \quad C_{24} = 0, \quad G_4 = 0$$

$$C_{15} = 0, \quad C_{25} = 1, \quad G_5 = 0$$
(12)

Finally, using the Maggi equations (4) and dependences (6)–(12) we received dynamic motion equations for the mobile robot  $\operatorname{AmigoBot}^{TM}$  in the form generated by the  $\operatorname{Maple}^{TM}$  program environment. Presented mobile robot has 2 DOF, it means in order to describe it's dynamic we need two equations.

$$\frac{(r^{2}m_{4}l_{1}^{2} + 4r^{2}m_{1}l_{1}^{2} + 2r^{2}Ix_{1} + 4Iz_{1}l_{1}^{2} + r^{2}Iz_{4} + r^{2}m_{4}l_{2}^{2})}{4l_{1}^{2}}\ddot{\alpha}_{1} \\
+ \frac{(r^{2}m_{4}l_{1}^{2} - r^{2}m_{4}l_{2}^{2} - 2r^{2}Ix_{1} - r^{2}Iz_{4})}{4l_{1}^{2}}\ddot{\alpha}_{2} \\
- \frac{r^{3}m_{4}l_{2}}{4l_{1}^{2}}\dot{\alpha}_{1}\dot{\alpha}_{2} + \frac{r^{3}m_{4}l_{2}}{4l_{1}^{2}}\dot{\alpha}_{2}^{2} = M_{1} - N_{1}f_{1}sgn(\dot{\alpha}_{1}) \\
\frac{(r^{2}m_{4}l_{1}^{2} + 4r^{2}m_{1}l_{1}^{2} + 2r^{2}Ix_{1} + 4Iz_{1}l_{1}^{2} + r^{2}Iz_{4} + r^{2}m_{4}l_{2}^{2})}{4l_{1}^{2}}\ddot{\alpha}_{2} \tag{13} \\
+ \frac{(r^{2}m_{4}l_{1}^{2} - r^{2}m_{4}l_{2}^{2} - 2r^{2}Ix_{1} - r^{2}Iz_{4})}{4l_{1}^{2}}\ddot{\alpha}_{1} + \\
- \frac{r^{3}m_{4}l_{2}}{4l_{1}^{2}}\dot{\alpha}_{1}\dot{\alpha}_{2} + \frac{r^{3}m_{4}l_{2}}{4l_{1}^{2}}\dot{\alpha}_{1}^{2} = M_{2} - N_{2}f_{2}sgn(\dot{\alpha}_{2})$$

After basic transformations and substitutions the form of dynamic motion equations for our model have been presented as follow:

$$a_{2}\ddot{\alpha}_{2} + a_{1}\ddot{\alpha}_{1} + a_{3}\dot{\alpha}_{2}^{2} - a_{3}\dot{\alpha}_{1}\dot{\alpha}_{2} = M_{1} - a_{4}\mathrm{sgn}(\dot{\alpha}_{1})$$
  
$$a_{2}\ddot{\alpha}_{1} + a_{1}\ddot{\alpha}_{2} + a_{3}\dot{\alpha}_{1}^{2} - a_{3}\dot{\alpha}_{1}\dot{\alpha}_{2} = M_{2} - a_{5}\mathrm{sgn}(\dot{\alpha}_{2})$$
(14)

where

$$a_{1} = \frac{1}{4} \frac{(r^{2}m_{4}l_{1}^{2} + 4r^{2}m_{1}l_{1}^{2} + 2r^{2}Ix_{1} + 4Iz_{1}l_{1}^{2} + r^{2}Iz_{4} + r^{2}m_{4}l_{2}^{2})}{l_{1}^{2}}$$

$$a_{2} = \frac{1}{4} \frac{(r^{2}m_{4}l_{1}^{2} - r^{2}m_{4}l_{2}^{2} - 2r^{2}Ix_{1} - r^{2}Iz_{4})}{l_{1}^{2}}$$

$$a_{3} = \frac{1}{4} \frac{r^{3}m_{4}l_{2}}{l_{1}^{2}} \qquad a_{4} = N_{1}f_{1} \qquad a_{5} = N_{2}f_{2}$$
(15)

however:

 $m_2 = m_1, m_4$  – mass of particular unit element.  $Ix_2 = Ix_1, Iz_2 = Iz_1, Iz_4$  – moments of inertia in relation to particular axis.  $M_1, M_2$  – drive moments.

 $N_1$ ,  $N_2$  – the pressure forces of particular wheels.

 $f_1$ ,  $f_2$  – rolling friction factors of wheels 1 and 2.

 $\alpha_1$ ,  $\alpha_2$  – angles of rotation (wheels 1 and 2).

The next step in the process of the precise description of the wheeled mobile robot mathematical model is to present dynamic motion equations in state space form. Taking into consideration state variables as follows:

$$\alpha_1 = x_1, \quad \dot{\alpha}_1 = \dot{x}_1 = x_2, \quad \alpha_2 = x_3, \quad \dot{\alpha}_2 = \dot{x}_3 = x_4$$
(16)

On the base of assumption (16) the dynamic motion equations (14) have been written in the state space form as follow:

$$\dot{x} = Ax + B[f(x,a) + G(x,a)u] \tag{17}$$

where:

where

 $l_1$  [m]

0.12

 $l_3$  [m]

0.08

 $l_4$  [m]

0.03

$$G_{31} = \frac{a_1}{a_1^2 - a_2^2} \qquad G_{32} = -\frac{a_2}{a_1^2 - a_2^2}$$

$$G_{41} = -\frac{a_2}{a_1^2 - a_2^2} \qquad G_{42} = \frac{a_1}{a_1^2 - a_2^2}$$

$$f_3 = \frac{a_2a_5\text{sgn}(x_4) + a_2a_3x_2^2 - a_2a_3x_2x_4 + a_1a_3x_2x_4 - a_1a_3x_4^2 - a_1a_4\text{sgn}(x_2)}{a_1^2 - a_2^2}$$

$$f_4 = \frac{-a_1a_5\text{sgn}(x_4) - a_1a_3x_2^2 + a_1a_3x_2x_4 - a_2a_3x_2x_4 + a_2a_3x_4^2 - a_2a_4\text{sgn}(x_2)}{a_1^2 - a_2^2}$$

Vector u stands for the driving moments  $M_1$  and  $M_2$ , which can be measured or which can be generated on the basis of dynamic motion equations. The form of the dynamic motion equations for wheeled mobile robot as in (17) allows for identification of the mathematical model of the robot. The following movement model has been assumed: starting, driving straight, driving on circular curve with turning axis of frame  $\beta$  and radius of wheels r, braking. The robot moves with the velocity  $v_a=0,2$  [m/s] (velocity of point A in Fig. 1.).

The time courses of driving moments for this movement model and construction data included in Tab. 1 and Tab 2 have been presented in Fig. 3.

<b>Table 2</b> The construction data of the AmigoBot <sup><math>TM</math></sup> robot based on dynamics					
$l_2$ [m]	$m_1$ [kg]	$m_2  [\mathrm{kg}]$	$m_3$ [kg]	$m_4$ [kg]	$Ix_1[kgm^2]$
0.06	0.65	0.65	0.4	2.25	0.01
$Ix_2[kgm^2]$	$Ix_3[kgm^2]$	$Iz_1[\mathrm{kgm}^2]$	$Iz_2[\mathrm{kgm}^2]$	$Iz_3[kgm^2]$	$Iz_4[kgm^2]$
0.01	0.004	0.025	0.025	0.0018	0.06
$N_1$ [N]	$N_2$ [N]	$N_3$ [N]	$f_1$ [m]	$f_2$ [m]	$f_3$ [m]
12.5	12.5	11.7	0.01	0.01	0.001

**Table 1** The construction data of the  $\operatorname{AmigoBot}^{TM}$  robot based on kinematics  $l_5$  m

0.12

 $r_1$  m

0.05

 $r_2 [m]$ 

0.05

 $r_3 [m]$ 

0.03

As a representation of mobile robot  $\operatorname{AmigoBot}^{TM}$  robot kinematics parameters representing rotation and angular velocity of particular wheels have been measured and some of those parameters have been shown in Fig. 4.

The measurements have been taken on real object in laboratory environment.



Figure 3 Time courses of driving moments  $M_{1-}$  blue and  $M_{2-}$  red



Figure 4 Time courses of angular velocity of particular wheels ( $\dot{\alpha}_{1}$ - blue,  $\dot{\alpha}_{1}$ - red)

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# 3. Identification of the dynamic motion equations of the $AmigoBot^{TM}$

Since dependence f(x, a) does not have linear representation due to the parameters, the system which is subject to identification can be presented in the form of a parallel structure [3,4,5]:

$$\dot{\hat{x}} = A\hat{x} + B[\hat{f}(\hat{x},\hat{a}) + G(a)u] + K\tilde{x}$$
(19)

where vector  $\hat{x}$  is the estimator of the state vector x,  $\hat{f}(\hat{x}, \hat{a})$  is the estimator of nonlinear functions appearing in equation (17). In equation (19) vectors u and  $\tilde{x}$  are not simultaneously known. After computational algorithm, parameters which appear in dynamic equations have been received. It is an example of off-line identification. Taking into consideration the estimation error of the state vector in the form  $\tilde{x} = x - \hat{x}$  and subtracting equation (19) from equation (17), the description of the identification unit in the errors space has been received:

$$\dot{\tilde{x}} = A_H \tilde{x} + B[\tilde{f}(x, \hat{x}, a, \hat{a}) + G(a)u]$$
(20)

where  $A_H = A - K$ , and matrix K has been so projected that characteristic equation of matrix  $A_H$  should be strictly stable.

$$\tilde{f}(x, \hat{x}, a, \hat{a}) = f(x, a) - \hat{f}(\hat{x}, \hat{a})$$
(21)

Where as in parallel structure vector  $\hat{x}$  is the estimator of the state vector x,  $\hat{f}(\hat{x}, \hat{a})$  is the estimator of non–linear functions appearing in equation (17).



Figure 5 Fuzzy logic system

Dependence (21) describes the differences between the algebraic expressions appearing in matrix f(x, a) and estimator of the non-linear functions matrix. In order to solve the presented identification problem fuzzy logic has been used. In the presented identification problem, the estimator of non-linear functions appearing

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Figure 6 Fuzzy logic interference system f31

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Figure 7 Fuzzy logic interference system f41



Figure 8 Membership function for first input of the system



 ${\bf Figure} ~{\bf 9} ~{\rm Membership} ~{\rm function} ~{\rm for} ~{\rm second} ~{\rm input}$ 

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 ${\bf Figure \ 10} \ {\rm Rules \ editor \ for \ fuzzy \ systems}$ 



 ${\bf Figure \ 11} \ {\rm Fuzzy \ logic \ system \ output}$ 

in matrix f(x, a) (18) has been subject to optimization in order to fit the simulation result to the behaviour of the real object represented by the measured kinematics parameters illustrated in Fig. 4. To create fuzzy logic interference systems, ready toolbox (*fuzzy logic toolbox*) available in the Matlab<sup>TM</sup> environment has been used Two fuzzy logic interference systems have been used since in matrix f(x, a) there are two functions for approximation, while the remaining equal 0.

These fuzzy logic systems called *Fuzzy Logic* f 31 and *Fuzzy Logic* f 41 have been modelled in Matlab<sup>TM</sup>–Simulink environment. The way of modelling has been presented in Fig. 5.

The fuzzy logic symbol presented in the upper part of Fig. 5. has been used in further part of this paper as a representation of this system [4,5]. With the use of fuzzy logic editor two fuzzy logic interference systems f31 and f41 have been created. Fuzzy logic system f31 has been presented in Fig. 6.

In the phase of designing the fuzzy logic systems, the model of Takagi–Sugeno type has been used. As an input for f31 two angular velocities  $\dot{\alpha}_1, \dot{\alpha}_2$  of particular wheels have been assumed  $(dalpha_1 \text{ and } dalpha_2 \text{ in Fig. 6})$ . The f31 system has a rule base and one output. The problem of modelling of these elements will be discussed in the further part of this paper. The fuzzy logic system f41 has been modelled in the same way as f31 system and has been presented in Fig. 7.

We are able to describe fuzzy systems in such a way that defuzzification block transforms input space in the form  $X = [\dot{\alpha}_{1a}, \dot{\alpha}_{1b}] \times [\dot{\alpha}_{2a}, \dot{\alpha}_{2b}] \subset \mathbb{R}^n$  into fuzzy set, characterized by membership function  $\mu_A(x) : X \to [0, 1]$ , i.e. assigns the level of membership to particular fuzzy logic sets. In Fig. 8. and Fig. 9. the used membership functions in the form of Gauss function with the input ranges of variability resulting from the range of possible to reach velocities ( $\dot{\alpha}_1 \in [0, 30], \dot{\alpha}_2 \in [0, 30]$ ) of mobile robot have been presented.

The basic rules for the described model have been used in the form presented in Fig. 10. Since for the particular fuzzy system inputs, six membership functions have been used, the following manner of creating rules has been applied: each membership function from  $\dot{\alpha}_1$  input with each membership function from  $\dot{\alpha}_2$  input. As a result of such a rule base construction we receive 36 rules.



Figure 12 Parallel structure of  $AmigoBot^{TM}$  mobile robot state emulator



**Figure 13** The space vector estimator,  $\hat{\alpha}_1$  [rad] – black,  $\hat{\alpha}_2$  [rad] – green,  $\dot{\hat{\alpha}}_1$  [rad/s] – blue,  $\dot{\hat{\alpha}}_2$  [rad/s] – red



Figure 14 The estimation error of the state vector,  $\tilde{\alpha}_1$  [rad]– black,  $\tilde{\alpha}_2$  [rad]– green,  $\dot{\tilde{\alpha}}_1$  [rad/s] – blue,  $\dot{\tilde{\alpha}}_2$  [rad/s] – red

Such an operation has been applied to both systems (f31 and f41). In the algorithm used for computations function *fminbnd* has been applied. This function finds the minimum of a function of one variable within a fixed interval; it means that fminbnd(fun, x1, x2) returns a value x that is a local minimize of the function that is described in fun in the interval, x1 < x < x2. In Fig. 11. fuzzy logic set output has been presented. In the process of the defuzzification using weighted sum method we receive one *Fuzzy Logic* f31 and one *Fuzzy Logic* f41 (see Fig. 5) output variable.

Provided that we dispose of the complete state vector, we can accept the identification system in the form of state emulator in the parallel [4,5,6].

The emulator scheme mode in the Matlab<sup>TM</sup>–Simulink environment has been presented in Fig. 12. The parallel structure presented in Fig. 12. has been used to execute off–line identification. The result of computations in the form of the space vector estimator has been presented in Fig. 13 and 14. We received kinematics parameters and error.

The presented mode of dynamic motion equations identification with the use of fuzzy logic can be applied to structures of various types, among others to parallel structure and serial-parallel structure of a state emulator.

#### 4. Summary and conclusions

The presented identification structure with the use of fuzzy logic allowed for receiving state vector  $\hat{x}$  which is compatible with state vector x of the computational model of AmigoBot<sup>TM</sup> robot. Indicator of quality calculated as the integral of the sum of the square of errors of angular velocities of each wheel equals 0.139 for parallel structure. The presented fuzzy sets creation mode can be used for the identification of any mobile robots and in the process of identification of any signals. The presented offline identification mode may be used for learning fuzzy systems by neural networks. However, the learnt fuzzy systems can be used for online identification in control units e.g. in adaptive control.

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