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Research Article

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Ansys code applied to investigate the dynamics of composite sandwich beams

Abstract: A numerical analysis of the effect of temperature on the dynamics of the sandwich beam model with a viscoelastic core is presented. The beam under analysis was described with a standard rheological model. This solution allows one to study the effect of temperature on material strength properties. Calculations were performed with the Finite Element Method in the ANSYS software. The analysis of the results of the numerical calculations showed a significant influence of temperature on the strength properties of the model under test. The analysis confirmed damping properties of viscoelastic materials.

Keywords: composite, sandwich beam, free vibration, forced vibration, frequency, vibration amplitude, resonance

1 Introduction

A growing demand for lightweight and simultaneously durable materials, which can replace steel or other materials used in machine and device structures, gives rise to continuous development in this field of scientific investigations. Such characteristics are met by structural composite materials. Due to their mechanical properties, composite materials are used in the aerospace, marine, automotive and space industries. They are characterized by high resistance to electrochemical corrosion, high strength and rigidity. At the same time, they are significantly more lightweight than traditional construction materials.

Investigations on the dynamics of sandwich structures with a viscoelastic core had already begun in the 1950s. A damping analysis of the three-layer composite plate with a viscoelastic ply was carried out by Ross et al. [1]. Using the classic partial differential equation of undamped vi-

brations for a three-layer plate with a viscoelastic core, the authors obtained an equation for a damped motion. The equation was derived by recording the bending stiffness and the mass of the plate in a complex form. That study proved shear-based vibration damping methods were more effective and efficient than tensile-based methods. The method of dynamic analysis presented in [1], referred to as the Ross-Kerwin-Unger (RKU) method, is still applicable. In the 2010s, Jones's manual [2] on damping in viscoelastic systems, where the RKU method was presented and discussed, was published. At the same time, Rao [3] and Martinez-Agirre et al. [4] also published studies, where the modeling of damping properties of a sandwich structure with the RKU method was carried out.

The results of investigations of the dynamics of sandwich structures in which continuous mathematical models were applied showed that particular attention should be paid to the modeling of viscoelastic properties of the core material. Initially, in the investigations on the dynamics of multilayered composites with a viscoelastic core, researchers used classic rheological models (e.g., [5]). However, the classic rheological models, described with integer-order differential equations, generate inaccuracies in the model as regards frequency. A slope of experimental frequency amplitude characteristics curves is then always slighter than the corresponding modeling curves. This was the main reason for using a fractional rheological model in the description of viscoelastic properties of the core. The first studies devoted to this way of modeling appeared in the 1980s (e.g., [6]), but its comprehensive description is to be found in Pritz's study [7].

In the first two decades of the 21st century, numerous studies devoted to the dynamics of various types of systems with a fractional rhetorical model were published; for instance, studies by Beda and Chevalier [8], Cortes and Elejabarrieta [9, 10], De Espindola et al. [11], Monje et al. [12], Ghanbari and Haeri [13], and a survey by Rossikhin and Shitikova [14]. Recently, results of the dynamics of axially moving sandwich structures in the temperature field attained with this model have been presented by Marynowski [15, 16].

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The development in numerical methods, especially the Finite Element Method (FEM), observed in the last decades of the 20th century, also resulted in their application in the investigations of mechanics of composite systems. One of the first studies in this field was conducted by Rao *et al.*, who published their results in 1997 [17]. Within the FEM, the authors determined eigenfrequencies and modal loss coefficients of viscoelastic facings.

Zhi *et al.* [18] conducted investigations on the sandwich composite with a honeycomb core reinforced with aluminum grids. Facings were made of carbon fiber. The authors of the study examined critical load, rigidity and energy absorption. With the FEM, they calculated the values of critical loads at which the model under test was subject to buckling.

Schneider *et al.* [19] presented the results of quasi-statistical and dynamic investigations on the properties of corrugated-core sandwich panels. The models used for the tests were compressed. The destruction mechanism during the compression test of the composite model within the FEM analysis was also demonstrated by Zhou J. *et al.* [20].

The FEM used for the dynamics analysis of the laminate with a sandwich facing, where a high-order zigzag theory was applied, was presented by Kumar *et al.* [21]. A composite laminated plate with periodic fillers in the viscoelastic core material was analyzed by Zhou X. *et al.* [20, 22, 23].

Due to wide industrial applications of intelligent core composite structures today, the dynamic analysis of these structures is very important. The FEM was used to study the dynamics of sandwich beams and plates containing a magnetorheological fluid. The results of these studies are presented by Rajamohan *et al.* [24] and Yeh [25].

A survey of literature on this subject indicates that the dynamics of sandwich structures, which takes into account an influence of temperature, is relatively least recognized. On the other hand, a high demand for composite materials in the industry makes us suspect that companies conceal their research results in the name of trade secrets. A need for open investigations in this area follows from the above-mentioned aspects.

The present work is aimed at the analysis of an effect of temperature on the dynamics of the sandwich beam with a viscoelastic core. The dynamic analysis was carried out with the Ansys code. In the first part, two-variant computations of the dynamics of facings of the model under test were carried out. In the first variant, the facing was built of BEAM 188-type elements. In the second case, an 8-node SOLID 185-type element was used to generate a numerical model of the facing. For both the variants, calculations of the natural frequency of the facings were done. In the second stage, the facings were loaded with a harmonic si-nusoidal excitation force. The computational results were compared with the results obtained analytically and the experimental results published in [26] for a narrow range of temperature alternations. A solid three-dimensional numerical model of the entire sandwich beam with a viscoelastic core was then generated and calculations were carried out within a wide temperature range.

2 Effect of temperature and frequency on viscoelastic material dynamics

Multilayer composites with viscoelastic polymer cores are used in various branches of the industry and under different operating conditions. The dynamic properties of such composite structure elements are noticeably affected by the temperature and frequency of the dynamic excitation. The structure of the polymer cores consists of long, intertwined and crossed multiatomic chains. When polymers are homogeneous and isotropic, the stiffness and damping characteristics change with temperature. This is because the internal interaction that occurs during vibrations results in damping and energy dispersion. Cyclic loads acting on viscoelastic materials cause deformations of a periodic character due to the phase delay angle. To determine the stress-strain characteristics in such materials, the complex Young's modulus Ev, which depends on frequency, is determined:

$$E_v(\omega) = E(\omega)(1 + i\eta(\omega)) \tag{1}$$

where: $E(\omega)$ – Young's modulus at the reference temperature and frequency, $\eta(\omega)$ – loss coefficient.

Assuming that the Poisson's ratio of the viscoelastic material does not change with frequency, a complex Kirchhoff's modulus (shear modulus) can be similarly represented as:

$$G_v(\omega) = G(\omega)(1 + i\eta(\omega)) \tag{2}$$

For homogeneous isotropic polymers, Kirchhoff's modulus, Young's modulus as well as a complex bulk modulus are closely interrelated. In the case of polymers, values of complex moduli change very significantly with an increase in temperature, in a unique way for each polymer composition [2, 27]. Figure 1 shows temperature characteristics of a typical polymer.

Figure 1a shows that above the softening point *Ts* in the temperature transition region, the value of the shear modulus decreases considerably; whereas, the value of the
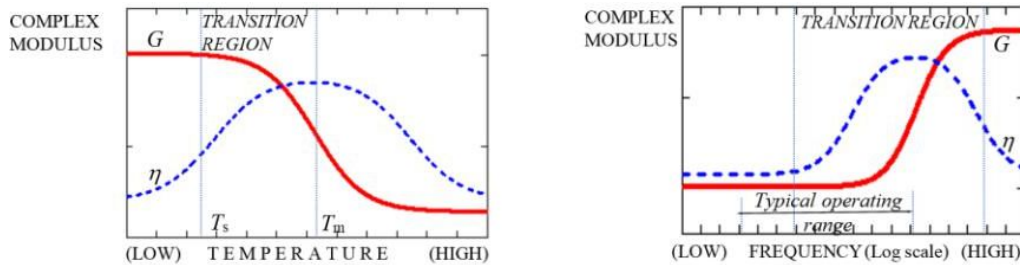
Figure 1: (a) Effect of temperature on complex modulus behavior. (b) Effect of frequency on complex modulus behavior [2, 27].

loss coefficient rises to its maximal value at the temperature *Tm*. Above this temperature, the value of the loss coefficient decreases. At temperatures above the transition region, the modulus value is low, and the material collapses as the temperature rises.

While the frequency effect is inconsiderable for typical metals, this influence is much stronger for polymers. The effect of frequency on complex moduli is an inverse of the influence of temperature. An increasing frequency acts similar to a decreasing temperature, but at a rate that differs very significantly, as illustrated in Figure 1b. Although the temperature can change by several hundred degrees for the material to fall within the transition region, the corresponding frequency change covers many orders of magnitude. In this range, the frequency can vary from 10−8 Hz to 108 Hz or more. In the low frequency range, shear modulus values and a loss coefficient increase slightly. Within the frequency transition range, a significant increase in the loss coefficient, which attains its maximal value and then decreases, can be seen. In this frequency range, the modulus value increases. Above the transition region, a further decrease in the loss coefficient value and a slight increase in the shear modulus, which then attains its maximal value, can be observed.

3 Object of numerical investigations

An analysis of dynamic properties is one of the basic stages in designing and constructing composite structures. The FEM can be applied for this purpose at the designing stage. An application of the FEM itself very rarely provides a complete and accurate description of dynamic properties of composite structures, especially when they are subjected to thermal loads. Under these conditions, it is reasonable to confront the FEM model of the designed structure with the results of experimental investigations. In this study, the results of experimental investigations of composite dynamic properties were taken from the literature.

A three-layer sandwich beam with a viscoelastic core of the length l = 250 mm, the width b = 10 mm and the thickness ($h_1 + h_2 + h_3$) = 6.205 mm is subject to numerical investigations. The tested model of a three-layer sandwich beam with a viscoelastic core is symmetrical with the following dimensions: thickness of the upper and lower lining $h_1 = h_3$ = 3 mm, tape thickness h_2 = 0.205 mm. A geometry of the system and a layer arrangement are shown in Figure 2. A composite beam of the same dimensions was subject to the experimental dynamic investigations described in [26].

Figure 2: Geometry of the system under consideration.

The material both the top and bottom facings were made of was a PA38/6060 aluminum alloy. Table 1 shows properties of the PA38/6060 aluminum alloy used to make the facings.

Table 1: Properties of the PA38/6060 facing material.

Property	Unit	Value
Density	[g/cm^3]	2.7
Poisson's ratio	-	0.33
Freezing point	[°C]	610
Melting point	[°C]	655
Heat conductivity	[W/mK]	200

The material the viscoelastic core was made of was a thin adhesive polyester tape. Table 2 shows characteristics of the TESA4965 tape used in the system under consideration.

Numerical calculations were performed with the FEM. The Ansys code was used in the computations. The calcula-

Table 2: Properties of the TESA4965 core material.

| Property	Unit	Value
Tape thickness	[µm]	205
Film thickness	[mm]	0.012
Density	[kg/m^3]	1.1
Maximal short-term temperature resistance	[°C]	80
Maximal long-term temperature resistance	[°C]	200

tions were made for a numerical model of the three-layer beam with a viscoelastic core and for its facings. In order to carry out the calculations, a model of the facing was generated for two cases. In the first case, the facing was made of a BEAM 188-type element. In the second case, an 8-node SOLID 185 element was used to build a numerical model of the facing. The numerical calculations were carried out to investigate an effect of temperature on the dynamics of the model under analysis. In the first stage, frequencies of free vibrations were calculated. In the second stage, the facings were loaded with a harmonic sinusoidal excitation force. The results of the calculations were compared with the results obtained analytically and the results of experimental investigations.

Then, a composite numerical model of a three-layer sandwich beam with a viscoelastic core was built and the calculations were repeated. First, the frequencies of free vibrations were calculated; second, the model was loaded with a harmonic sinusoidal excitation force. The investigated numerical model was divided into numerical elements with a length of 0.205 mm. For numerical calculations, all degrees of freedom were taken away in nodes of the lateral surface of one of the model ends. A division of the tested composite structure into finite elements is illustrated in Figure 3, and the numerical models of the facings used in the calculations are shown in Figure 4.
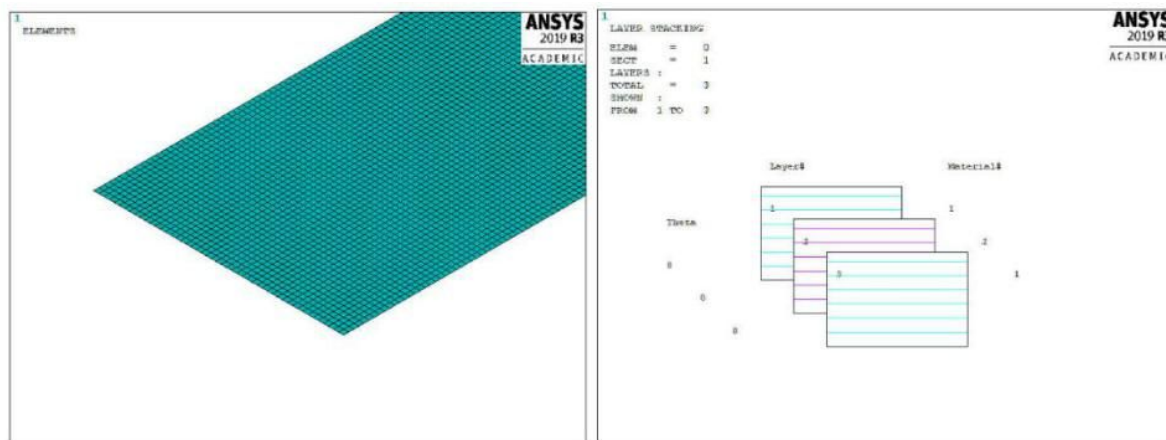
Figure 3: Division of the tested composite structure into finite elements.
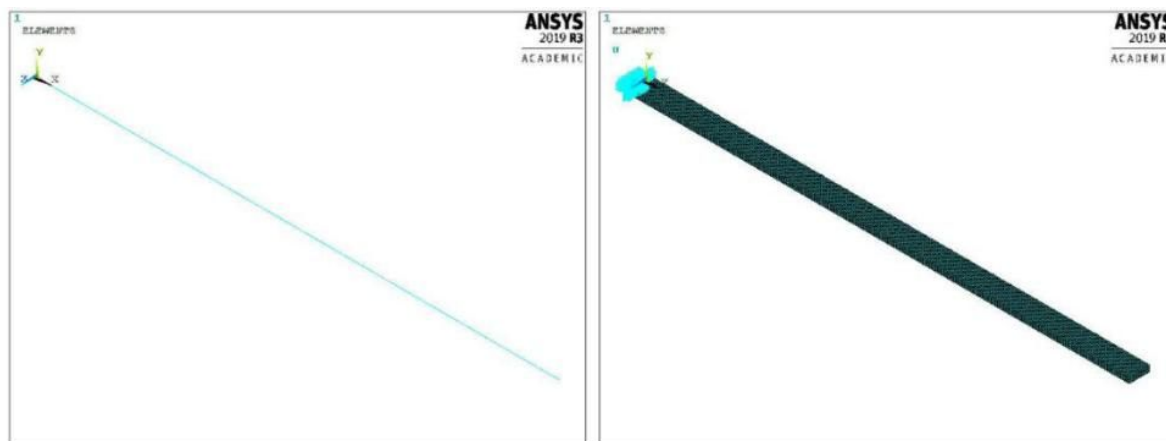
Figure 4: Numerical facing models: (a) division into BEAM 188 elements; (b) division into SOLID 185 elements.

A standard viscoelastic model, which is a simplification of the Prony's model [2, 27], was used to model the viscoelastic core. It is presented in Figure 5. The model consists of two branches connected parallelly; the first one is an elastic branch, the second – a spring with a damper – is connected in series.
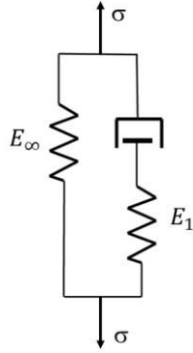

Figure 5: Standard structural model.

To define the viscoelastic material, Young's modulus (the so-called instantaneous modulus), a_1 – elasticity of the first viscoelastic branch, t_1 – time constant and η_1 – Prony's factor were introduced into the calculations. The constitutive equation of the model, presented in Figure 2, is shown below:

$$\sigma + t_1 \frac{d\sigma}{dt} = E_\infty \varepsilon + \frac{E_0 \eta_1}{E_1} \frac{d\varepsilon}{dt} \qquad (3)$$

where:

$$a_1 = \frac{E_1}{E_0} \qquad (4)$$

$$t_1 = \frac{\eta_1}{E_1} \qquad (5)$$

$$E_0 = E_1 + E_\infty \qquad (6)$$

The rheological parameters of the model used for the calculations are listed in Table 3.

**Table 3:** Standard model properties.

Properties of the standard model	Unit	Value
E_{1s}	[N/mm^2]	9.8656×10^2
E_{2s}	[N/mm^2]	2.87784×10^3
S	[Ns/mm^2]	2.3133862×10^5
a_1	-	0.255294483
t_1	hour	0.065236157
E_0	[N/mm^2]	3863.4
E_1	[N/mm^2]	986.56

4 Results of the calculations

The numerical calculations yielded values of frequencies of free vibrations and frequencies of forced vibrations excited with a harmonic sinusoidal force. The results of the numerical calculations were compared with the results of the analytical calculations and the experimental investigations presented in [26]. In Table 4, values of resonance vibration frequencies of a single facing for the ambient temperature are collected. An analysis of the results of the calculations obtained numerically and analytically showed a high compliance in both cases. A comparison of the numerical and analytical results to those obtained from the experiment showed that the resonance frequency value of the first mode was consistent. However, the values of the remaining frequencies obtained experimentally are slightly
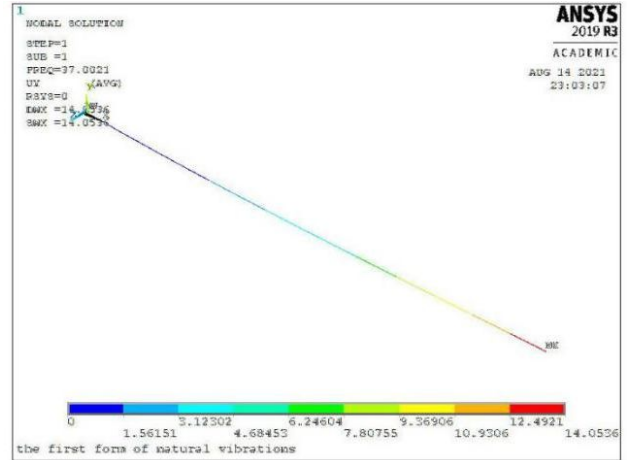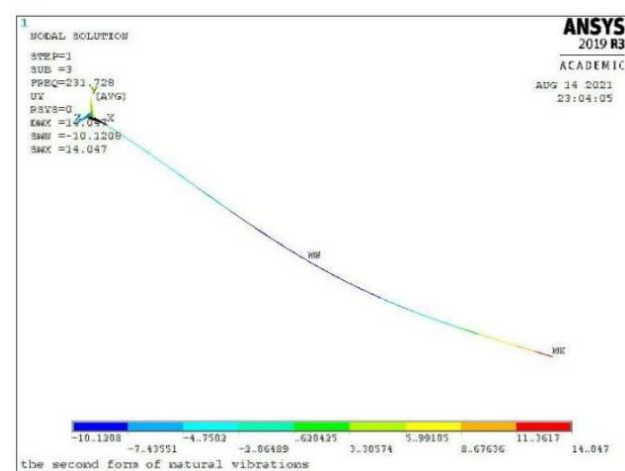

(a)
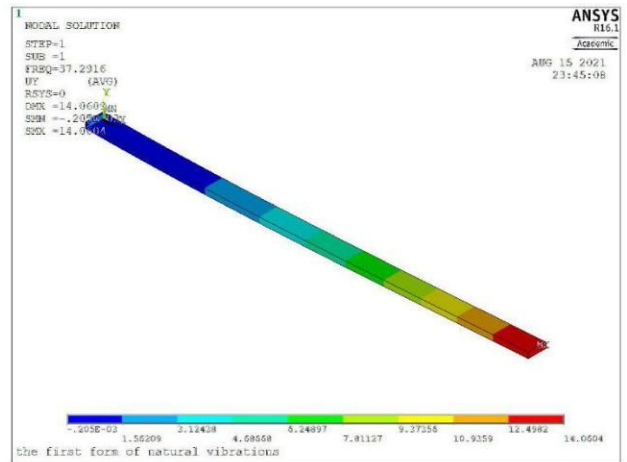
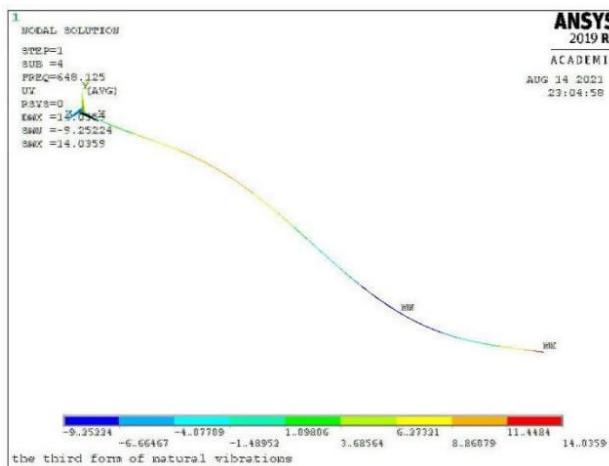**(b)**

Figure 6: First mode of free vibrations.

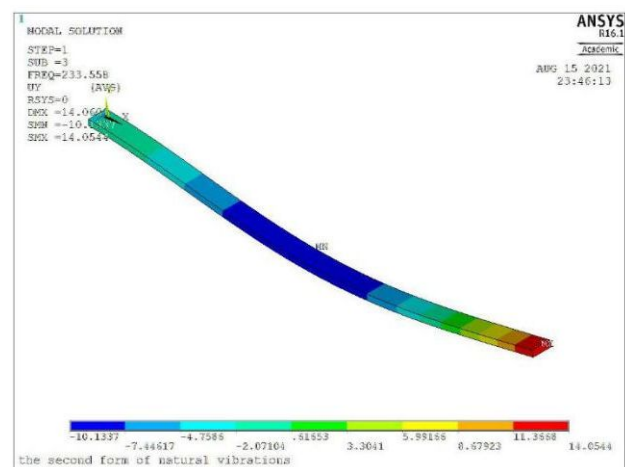**Table 4:** Free vibration frequencies of the facing in the ambient temperature.

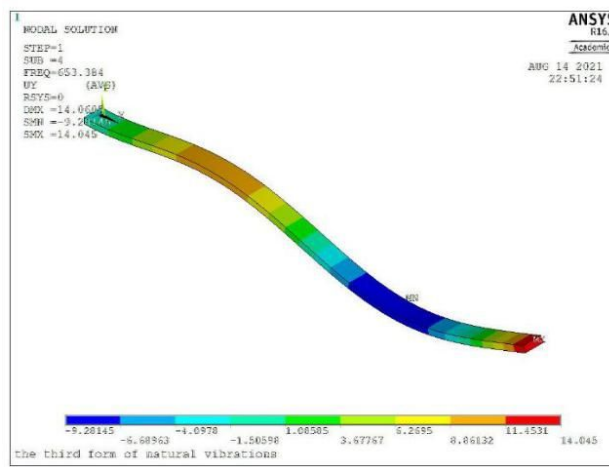| Mode	f [Hz]	ω [rad/s]	f [Hz]	ω [rad/s]	f [Hz]	ω [rad/s]
	BEAM 188		SOLID185		Analytical	
1	37	232	37	234	37	232
2	232	1456	233	1468	232	1456
3	648	4072	653	4105	649	4078
4	1268	7967	1278	8035	1273	7992
5	2092	13143	2110	13261	2545	15983

(a)

(a)

(b)

(b)

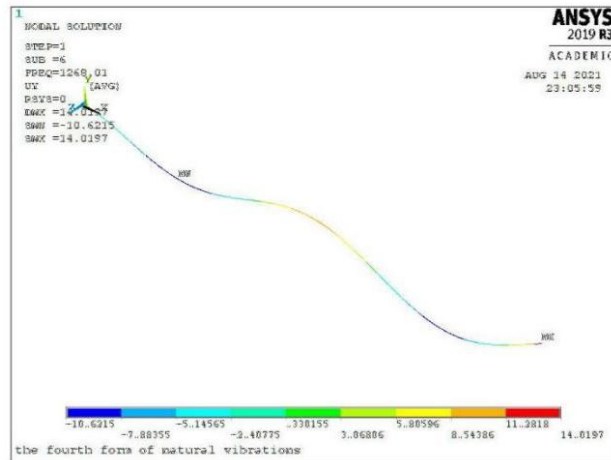**Figure 7:** Second mode of free vibrations.

Figure 8: Third mode of free vibrations.

lower than those obtained numerically and analytically. The results are gathered in Table 4.

Figures 6–10 show modes of free vibrations of the facing under analysis. Subsequent modes of free vibrations of the facing modeled with BEAM 188-type elements are depicted in Figures 6–10a, whereas Figures 6–10b show modes of free vibrations of the facing modeled with SOLID 185 elements.
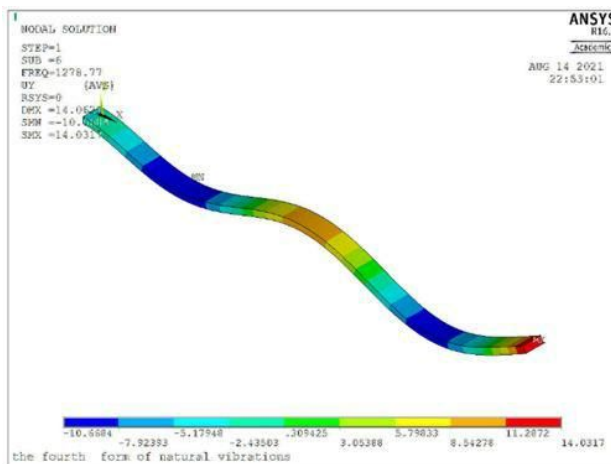
In the next stage, a dependence of the vibration amplitude on resonance frequencies was examined. Calculations
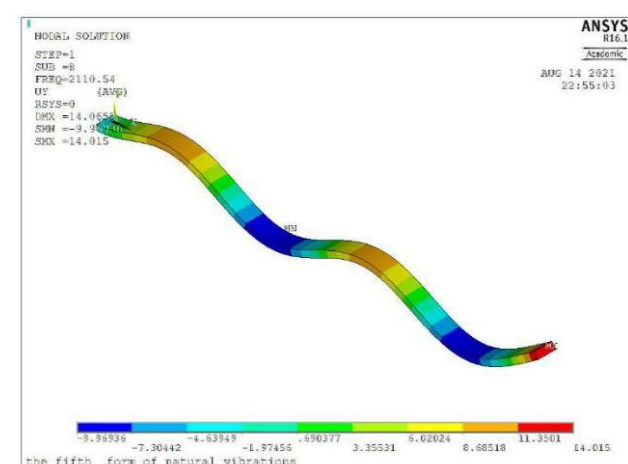
(a)
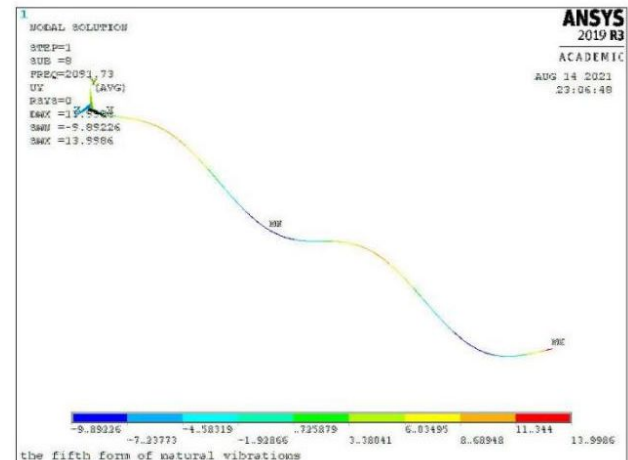
(b)

Figure 9: Fourth mode of free vibrations.

(a)

(b)

Figure 10: Fifth mode of free vibrations.

Table 5: Results of dynamic calculations of the facing under test.

| Mode | f | ω | f_r | ω_r |
	[Hz]	[rad/s]	[Hz]	[rad/s]
1	37	234	37	232
2	233	1468	231	1451
3	653	4105	630	3958
4	1278	8035	1240	7791
5	2110	13261	2070	13006

Figure 11: Diagram of resonance frequencies of the facing.

were made for the temperature equal to 20°C, 40°C and 60°C. An analysis of the results showed a tendency of the vibration amplitude to decrease with an increase in temperature. The results are summarized in Figure 11.

Table 5 presents the facing response to load with a harmonic sinusoidal force for the ambient temperature.
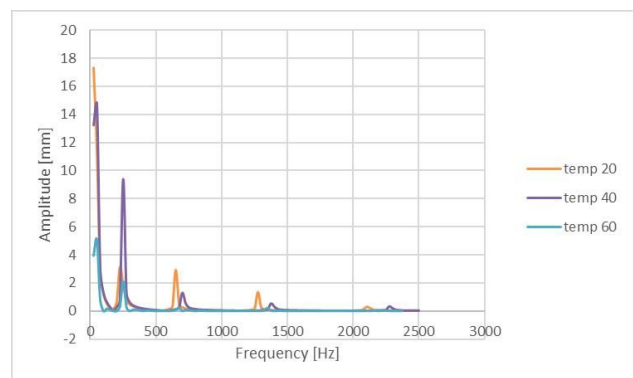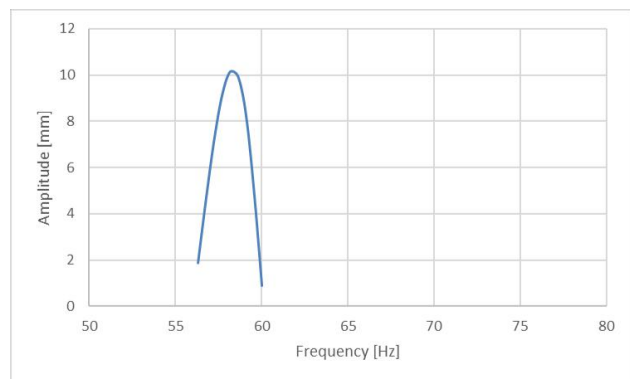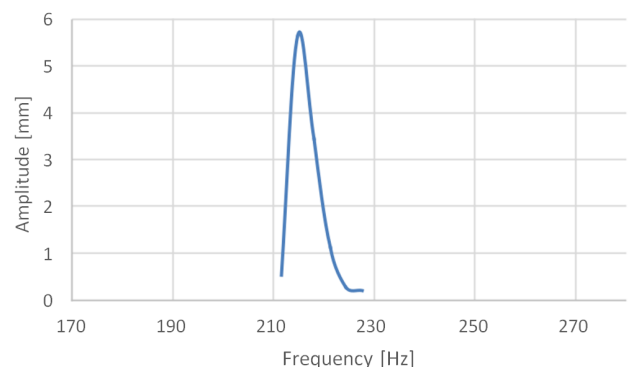
A response of the dynamics of the three-layer sandwich beam with a viscoelastic core to loading with a harmonic sinusoidal force is shown in Table 6. Calculations were made for different temperatures.

Table 6: Frequencies of free vibrations of the sandwich beam with a viscoelastic core obtained in the numerical calculations.

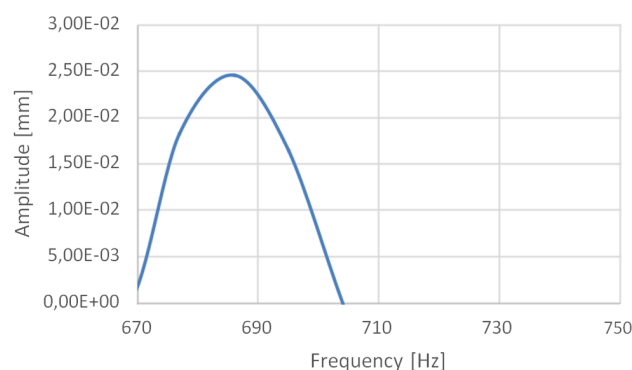| Mode	f	ω
	[Hz]	[rad/s]
1	58	364
2	214	1345
3	668	4197
4	1299	8168
5	2350	14765

Figure 12: Beam dynamic response – first resonance frequency.

Figure 13: Beam dynamic response – second resonance frequency.

Figure 14: Beam dynamic response – third resonance frequency.

the construction is one of the methods to control vibration and noise in the structure.

5 Discussion

Literature surveys have confirmed a high demand for lightweight materials with specific mechanical properties, such as high strength, rigidity, corrosion resistance and, simultaneously, other than conventional materials used in different structures. Composite materials are applied in various branches of the industry, e.g., aerospace, aviation, maritime, manufacturing of bicycle parts, etc.

The calculations were performed with the FEM, and the Ansys code was used to conduct the numerical analysis. The analysis of the results of the numerical calculations confirmed an effect of temperature on the dynamics of the model under test. The numerical calculations of natural frequencies were carried out for a single facing of the sandwich beam in the ambient temperature range. The computations were performed on a numerical model divided into BEAM 188 finite elements in the first case, and into SOLID 185 elements in the second case. As a result of the division into finite elements in the first case, 1000 elements and 1001 nodes were generated. In the second case, the model was divided into 130000 finite elements and 168336 nodes. The analysis of the numerical results obtained for a single facing showed a high consistency with the results of the analytical calculations and with the results of the experimental tests presented in [26].

A composite numerical model was built for the numerical analysis of a three-layer sandwich beam with a viscoelastic core. Four nodal SHELL 181 elements were used for the division into finite elements. A dynamic response to harmonic excitation was attained as a result of the calculations. The FEM allows one to perform numerical calcula-

A resonance diagram of the beam model is shown in Figure 12.

The numerical investigations confirmed a close relationship between temperature, vibration frequency of the tested model and, thus, material properties. An application of a core with viscoelastic properties confirmed its effect on the properties of the sandwich beam under investigation. Its operating conditions can be controlled, the dynamic characteristics can be affected, and the mechanical properties can be influenced via changes in temperature. In structures subjected to dynamic loads, damping is crucial. An application of materials with viscoelastic properties in

tions on models closest to reality, taking into account the strength properties of materials used in the construction, an influence of temperature. Nevertheless, they are not experimental investigations conducted on an actual structure. In addition, it was quite difficult to obtain material data and the values of the data used for the calculations were characterized by considerable approximation [29]. The analysis of the results of the numerical calculations showed that the amplitude of resonance vibrations decreased with an increase in temperature (Figures 5, 6, 7). As the temperature increased, the frequency of vibration decreased. The analysis of the obtained results confirmed damping properties of the core.

6 Conclusions

Numerical calculations or experimental investigations are not easy to conduct due to a lack of access to the properties of the composite materials used. A high demand for composite materials used in production causes that manufacturers protect the data behind the so-called trade secrets. Studies on scientific investigations have shown that there are a few works devoted to the dynamics of sandwich structures described with a fractional rheological model, which were verified by numerical calculations examining an effect of temperature on the dynamics of the tested model. Mechanical vibrations and their damping are extremely important in machines and devices. The investigations carried out showed an effect of working temperature on the dynamics of the tested model. Due to the use of a viscoelastic core, the dynamics of plates, beams, and so on can be controlled. It is one of the methods of vibration and noise control in the structure. It turns out that the damping layer has a significant impact on an application of the given element. This is crucial in engineering structures – for safe use of the structure and, thus, user's safety. It is sometimes difficult or impossible to obtain composite material data via experiments. It follows from the costs involved in carrying out the experiment or, for example, an inability to run tests at high temperatures.

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